طراحی اجزاء ۱

فصل ششم: شکست خستگی ناشی از بار متغیر

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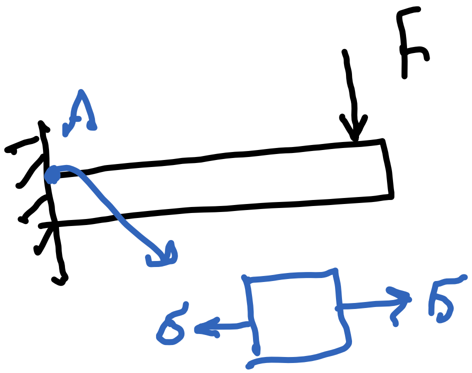
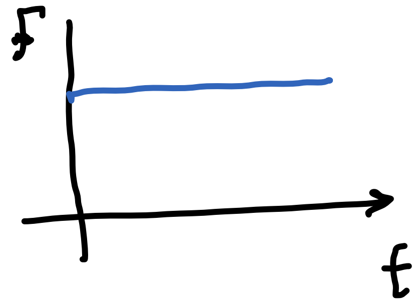
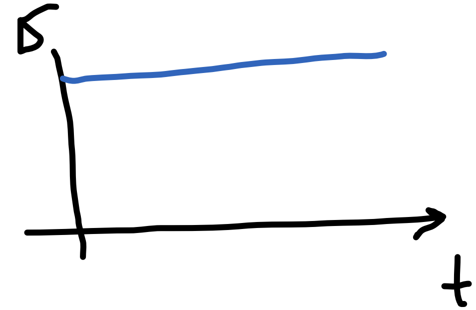
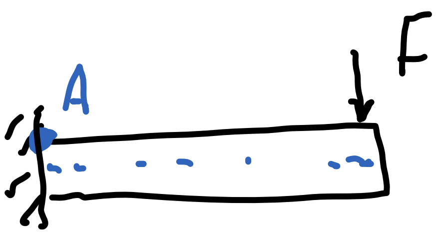
جلسه دهم

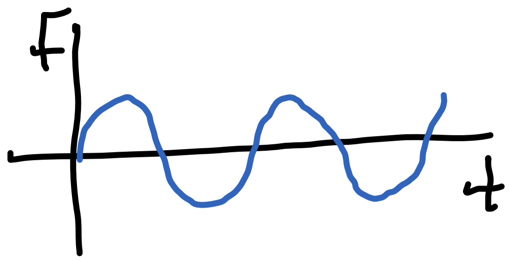
شکست خستگی ناشی از بار متغیر

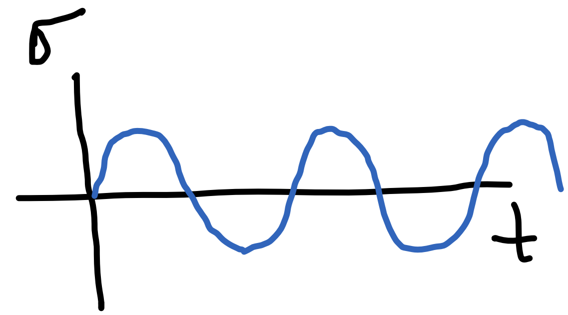


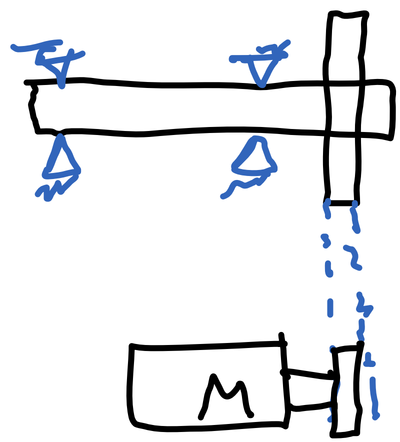


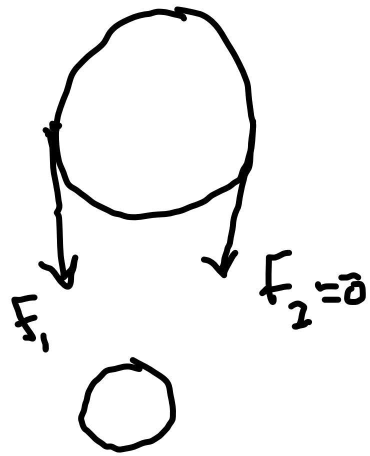


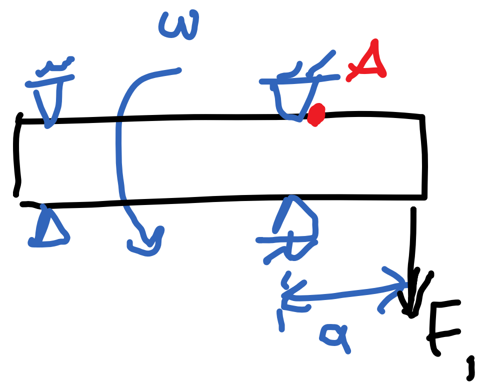






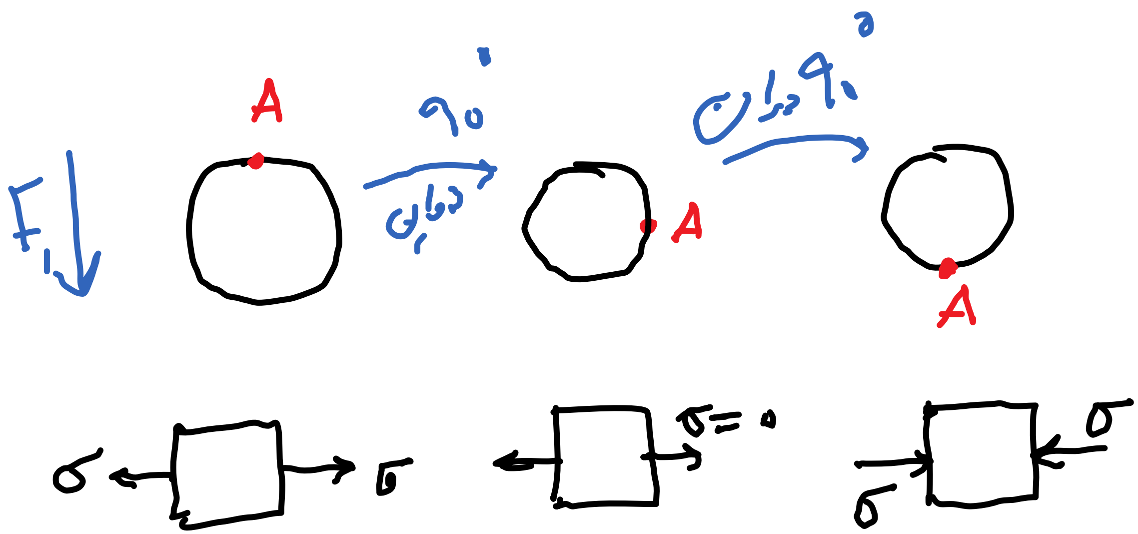


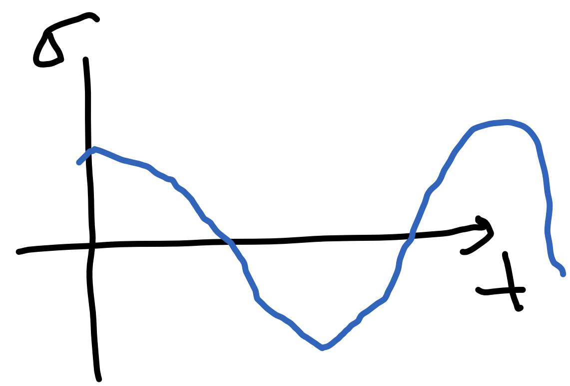




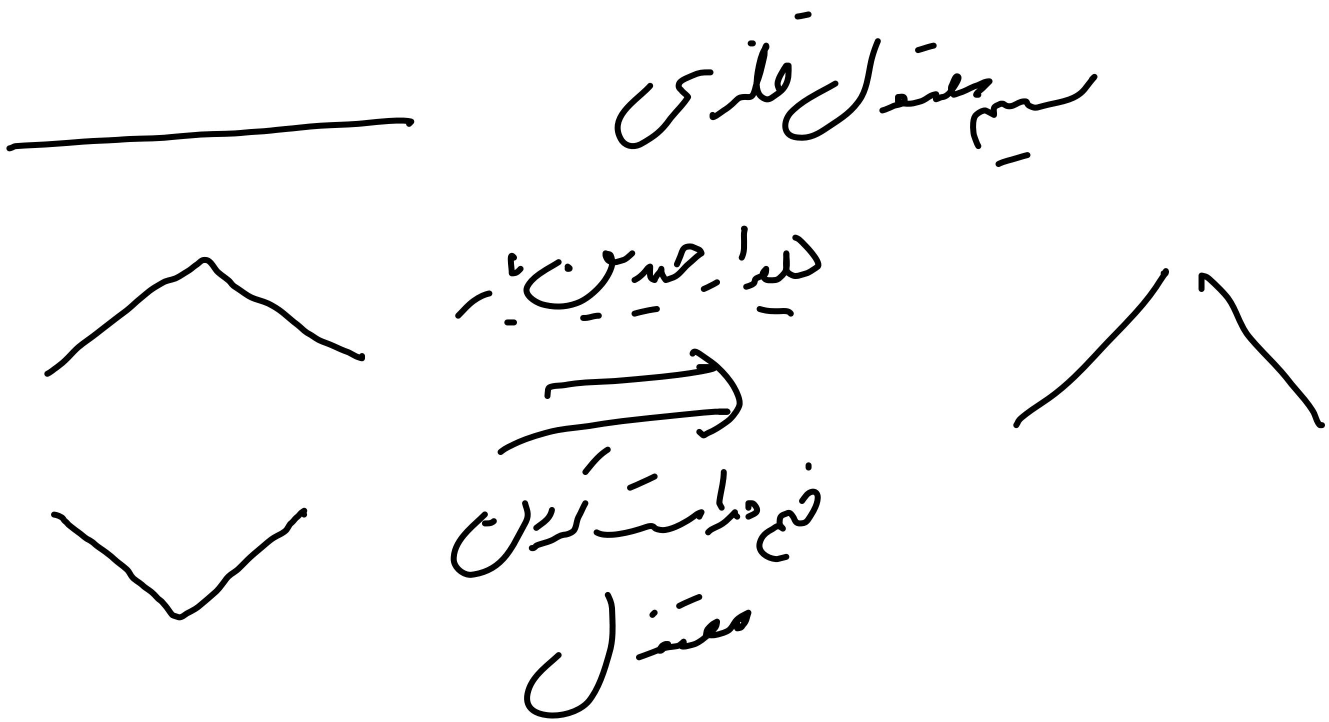
مقدار و جهت F_1 ثابت

شفت دوار





هرجایی که شفت یا تیر دوار داشته باشیم حتی اگر
نیروها ثابت باشد، تنشها نوسانی و متغیر هستند.



با اختراع ماشین بخار و شروع عهد صنعتی
قطعه‌های دوار به سرعت در ماشین آلات مختلف رواج
یافت

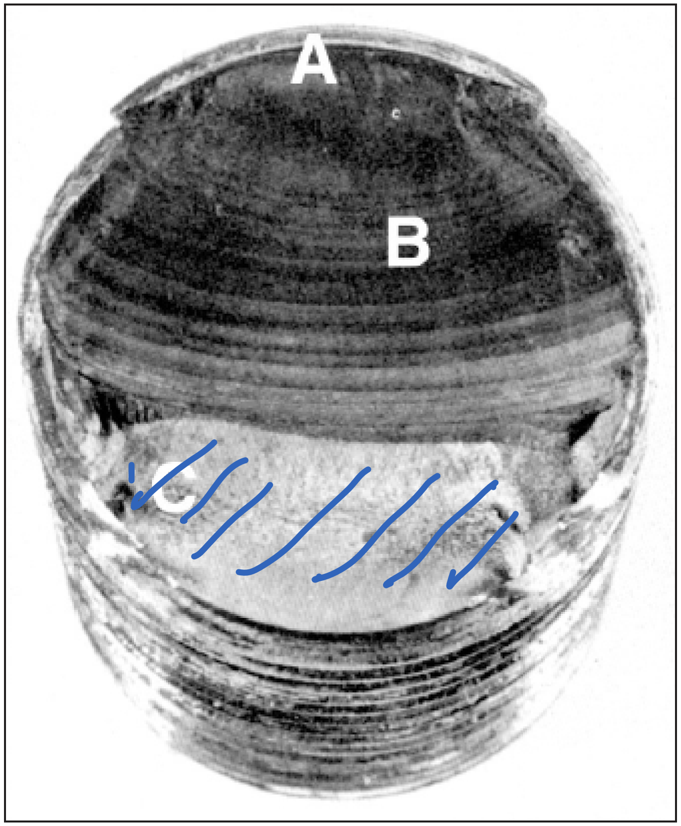
Historically, even as progress was made, dramatic failures continued to prove the need for continued study. A few notable examples that can be readily researched for details include the following: Versailles railroad axle (1842), Liberty ships (1943), multiple de Havilland Comet crashes (1954), Kielland oil platform collapse (1980), Aloha B737 accident (1988), DC10 Sioux City accident (1989), MD-88 Pensacola engine failure (1996), Eschede railway accident (1998), GE CF6 engine failure (2016).

به اثر تنش‌های نوسانی آرام آرام ترک‌های ریزی ایجاد می‌شود و با ادامه
بارگذاری آرام آرام ترک گسترش می‌یابد تا نهایتاً قطعه شکسته شود.

6–3 Crack Nucleation and Propagation

Fatigue failure is due to crack nucleation and propagation. A fatigue crack will initiate at a location that experiences repeated applications of locally high stress (and thus high strain). This is often at a discontinuity in the material, such as any of the following:

- Geometric changes in cross section, keyways, holes, and the like where stress concentrations occur, as discussed in Sections 3–13 and 5–2.
- Manufacturing imperfections such as stamp marks, tool marks, scratches, and burrs; poor joint design; improper assembly; and other fabrication faults.
- Composition of the material itself as processed by rolling, forging, casting, extrusion, drawing, heat treatment, and more. Microscopic and submicroscopic surface and subsurface discontinuities arise, such as inclusions of foreign material, alloy segregation, voids, hard precipitated particles, and crystal discontinuities.



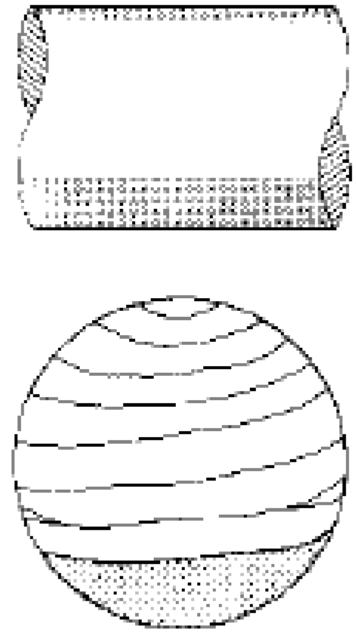
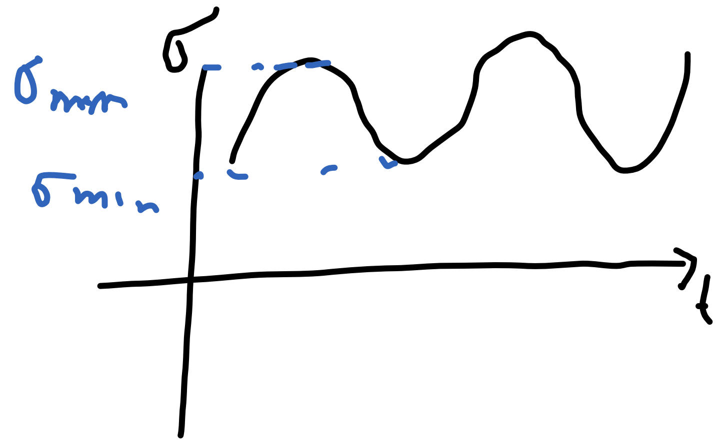
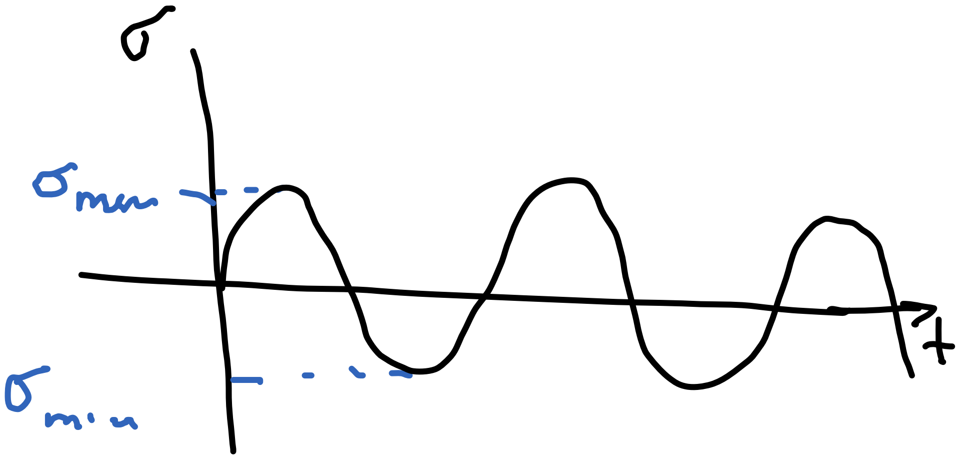


Figure 6–1

Fatigue failure of a bolt due to repeated unidirectional bending. The failure started at the thread root at *A*, propagated across most of the cross section shown by the beach marks at *B*, before final fast fracture at *C*. *(From*



تنش یک طرفه



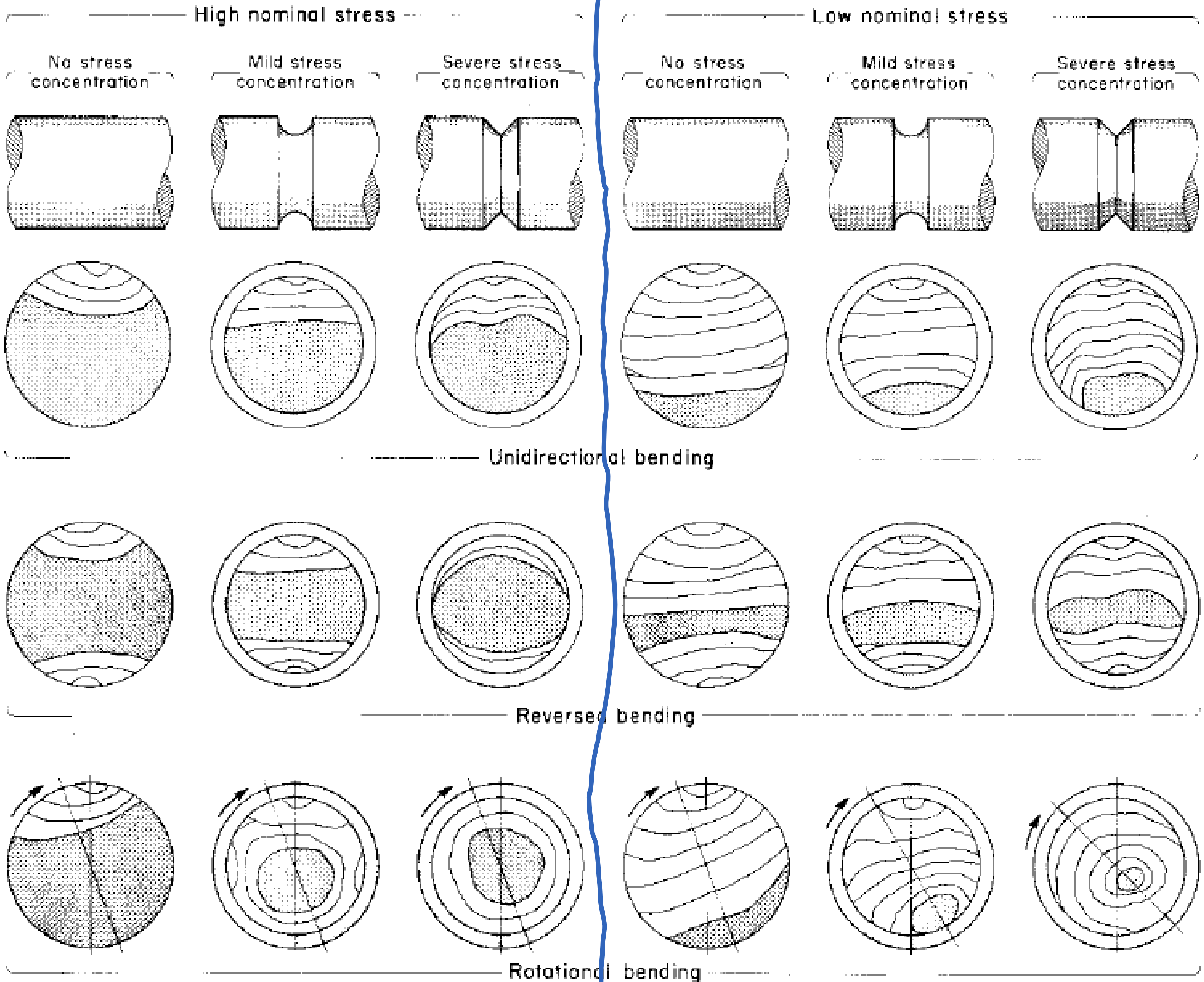
تنش دو طرفه

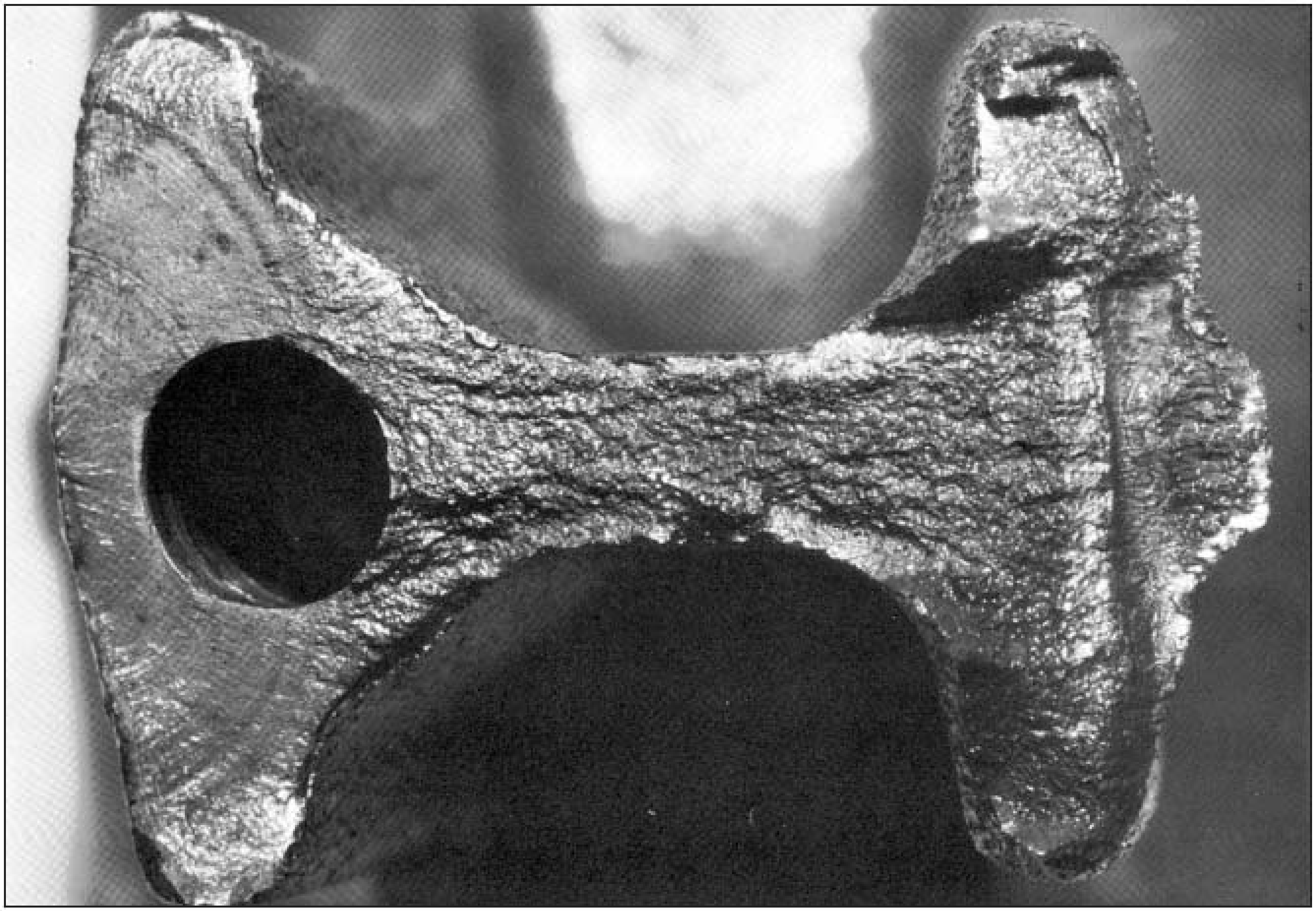
$$\sigma_{max} = |\sigma_{min}|$$

تنش معکوس شوند.

تنش پایین

تنش بالا





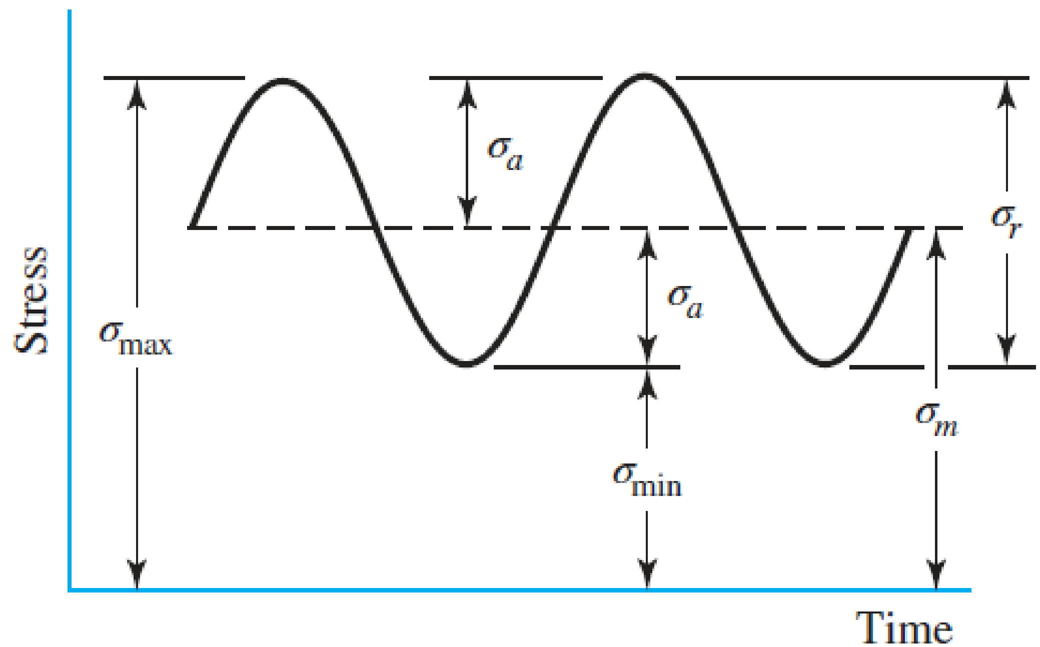
روشهای مختلف تحلیل عمر خستگی

- روش مکانیک شکست الاستیک خطی

- روش کرنش-عمر Strain-life

- روش تنش-عمر stress-life

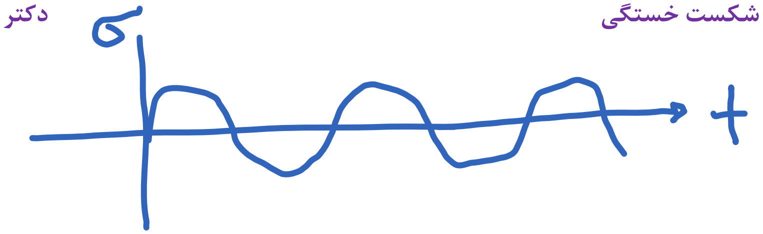
روش تنش- عمر و دیاگرام S-N

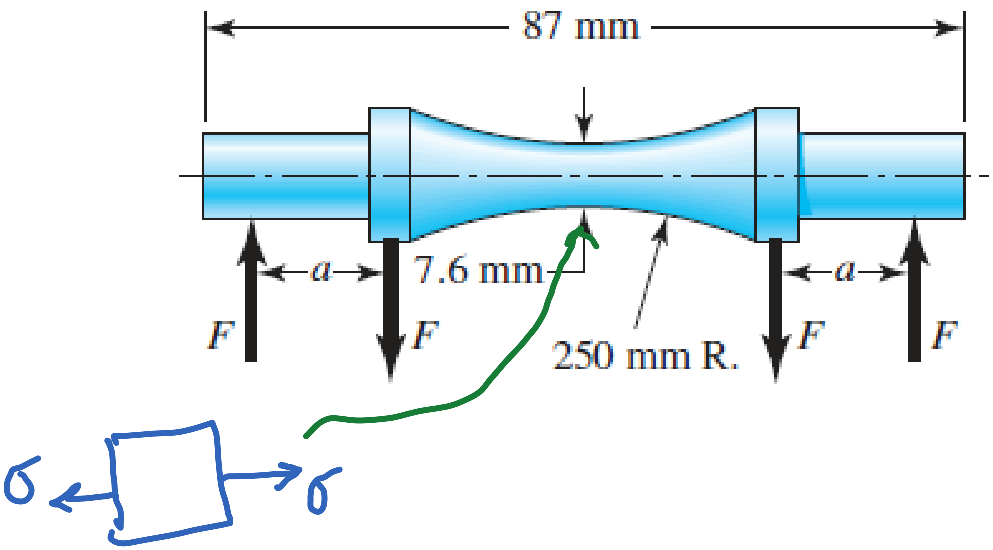


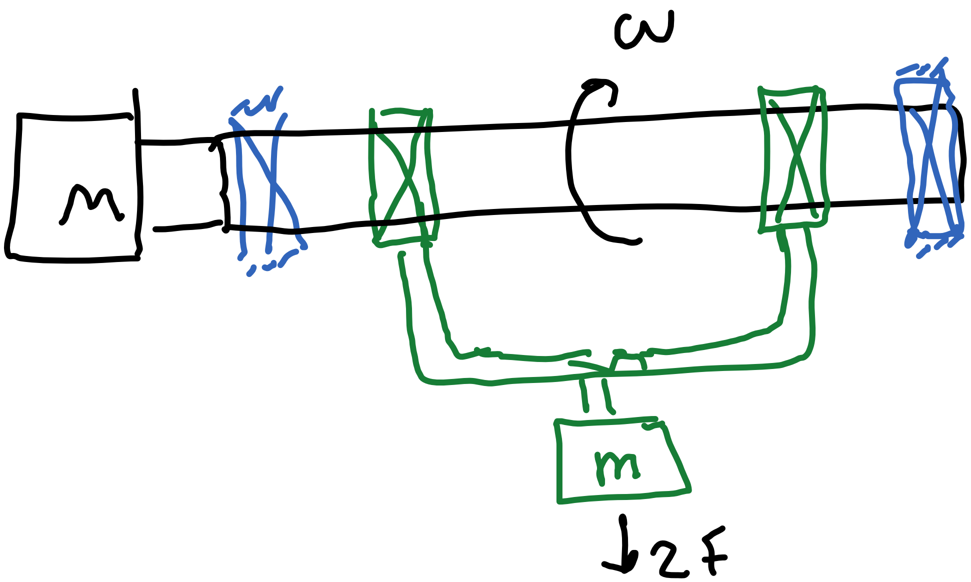
دامنه تنش $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$ $\sigma_r = 2\sigma_a$

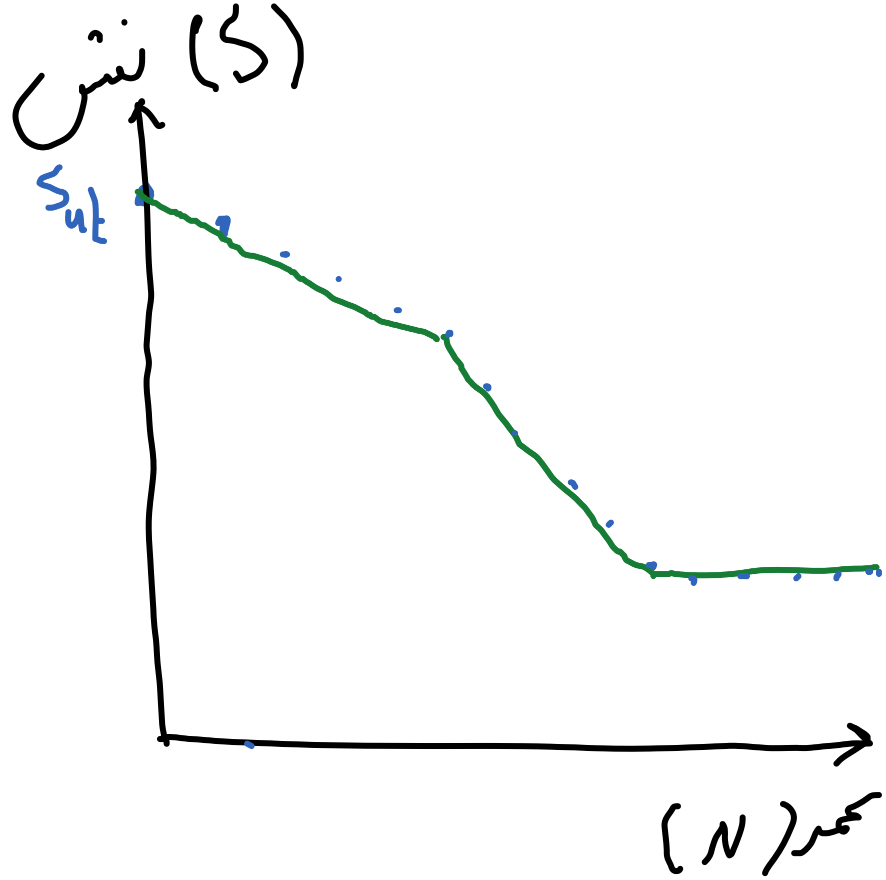
تنش میانگین یا متوسط $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

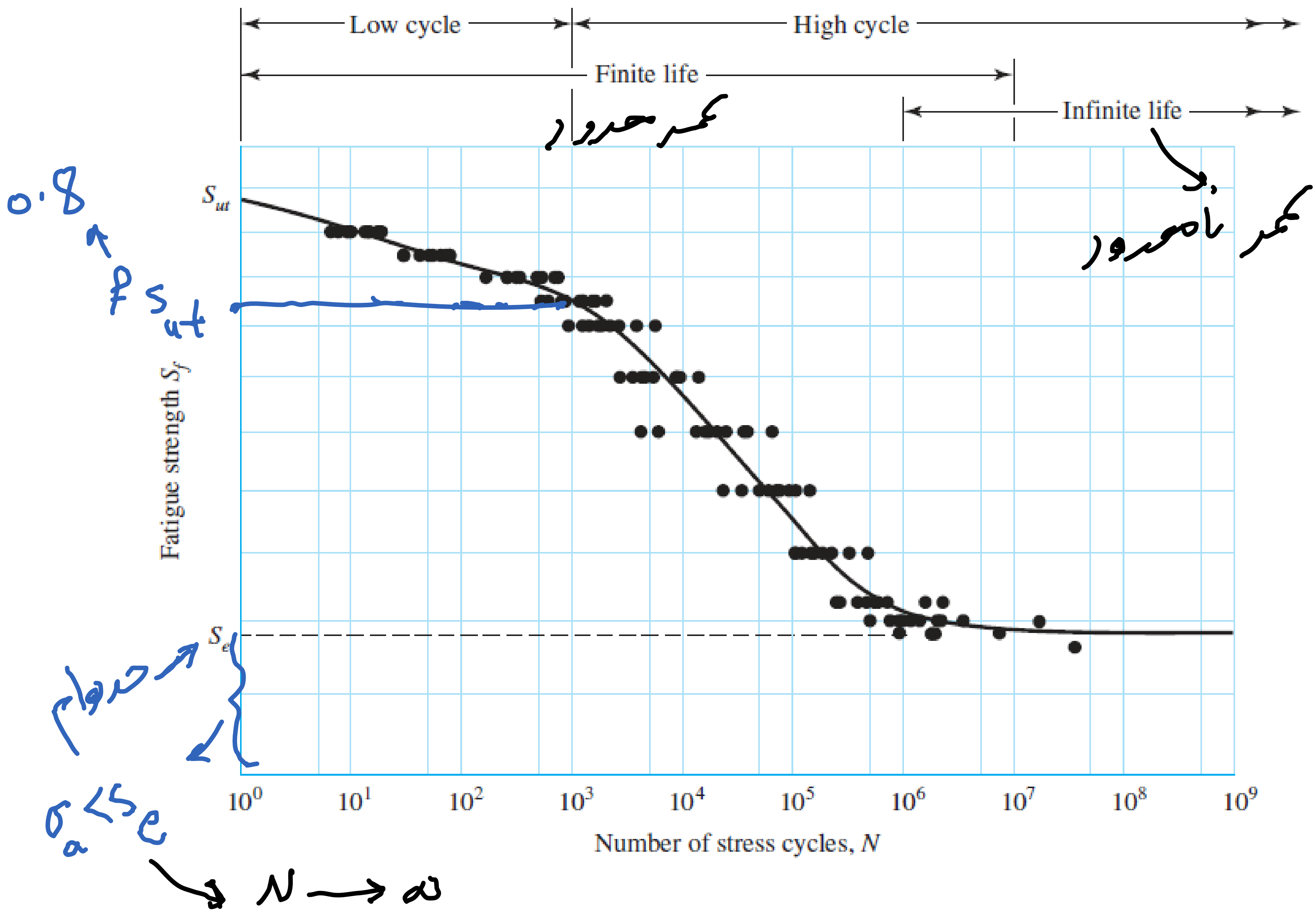
آزمایش تیر دوار
تحت تنش خالص










Low cycle
High cycle
Finite life
Infinite life
عمر محدود
عمر نامحدود
S_{ut}
$0.8 \; f S_{ut}$
Fatigue strength S_f
S_e
حد دوام
$\sigma_a < S_e \rightarrow N \rightarrow \infty$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9
Number of stress cycles, N

در حالت تنش‌های ثابت

$$\begin{cases} S_y & \text{استحکام تسلیم} \\ S_{ut} & \text{〃 ماکزیمم} \end{cases}$$

در حالت تنش متغیر

$$\begin{cases} S_e & \text{حد دوام} \end{cases}$$

endurance limit

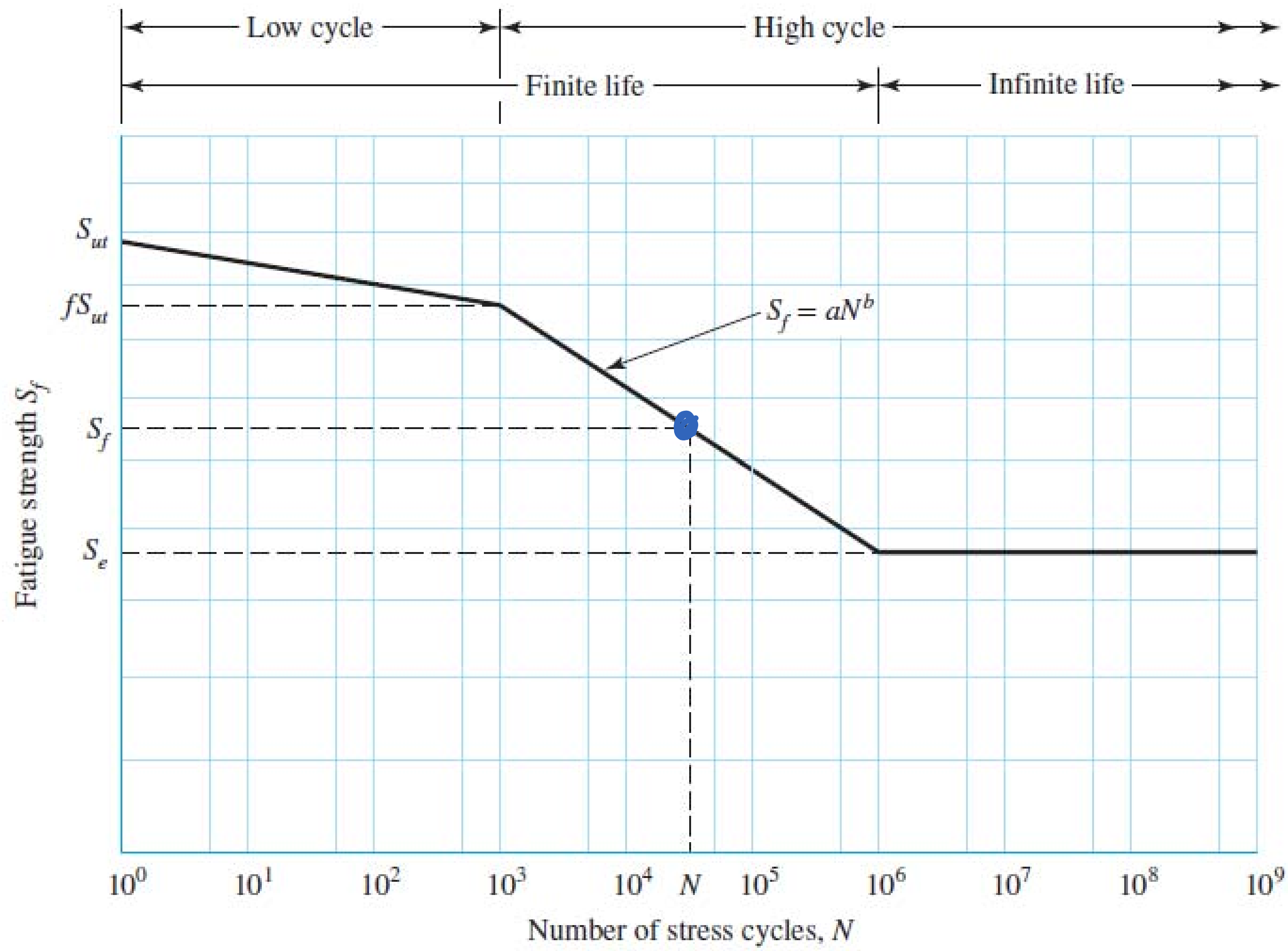


Figure 6–21

An idealized S-N diagram for steels.

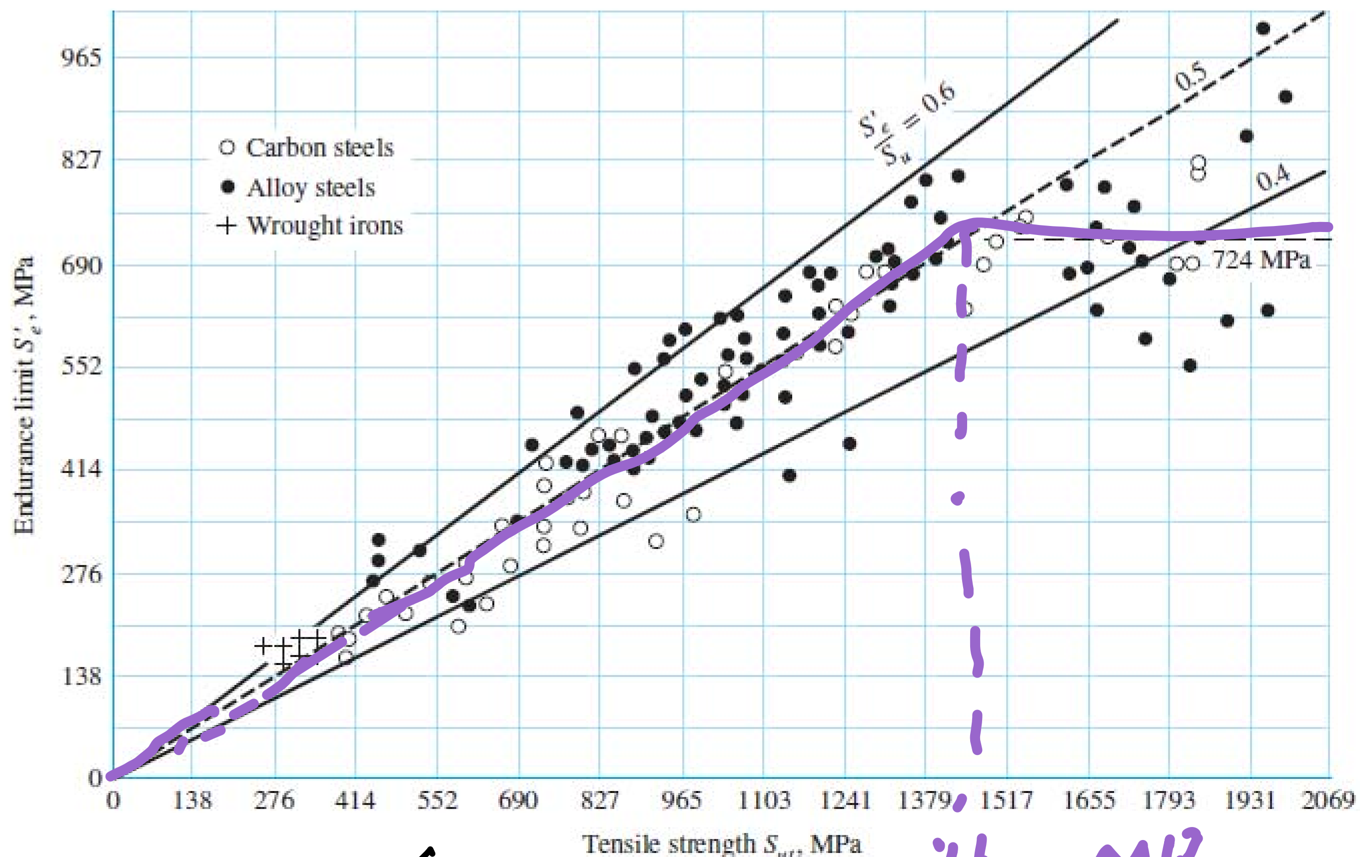
S_f استحکام خستگی

$$S_f = a N^b$$

جلسه یازدهم

تخمین حد دوام

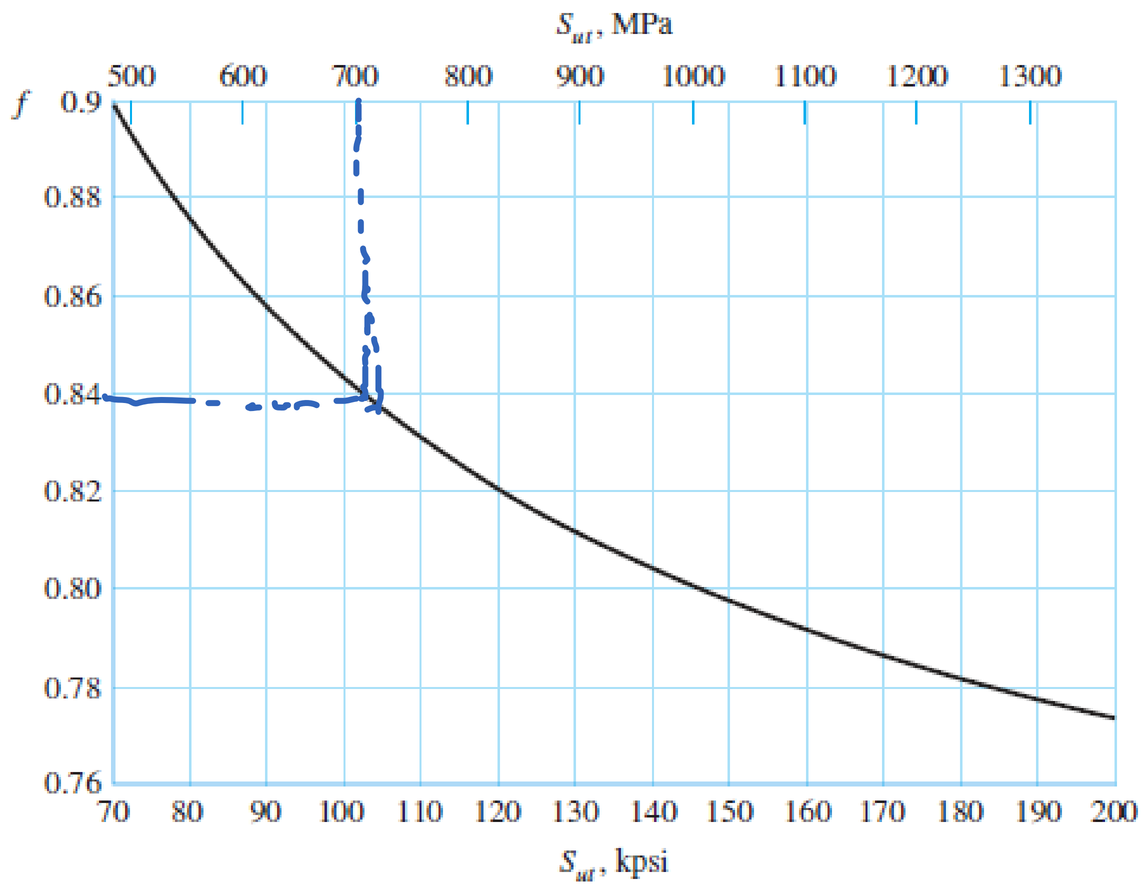
حد دوام



1400 MPa

$$S'_e = \begin{cases} 0.5\, S_{ut} & S_{ut} < 1400\ MPa \\ 700\ MPa & S_{ut} > 1400\ MPa \end{cases}$$

مقدار ضریب S_{ut} در سیکل 10^3 (f)



خط شکست بر چرخه $10^3 < N < 10^6$

$$S_f = a N^b$$

$$a = \frac{(fS_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$$

EXAMPLE 6–2

Given a 1050 HR steel, *estimate*
(*a*) the rotating-beam endurance limit at 10^6 cycles.
(*b*) the endurance strength of a polished rotating-beam specimen corresponding to 10^4 cycles to failure.
(*c*) the expected life of a polished rotating-beam specimen under a completely reversed stress of 379 MPa.

Solution

(*a*) From Table A–20, $S_{ut} = 620$ MPa. From Equation (6–10),

Answer

$$S'_e = 0.5(620) = 310 \text{ MPa}$$

(*b*) From Figure 6–23, or Equation (6–11), for $S_{ut} = 620$ MPa, $f \approx 0.86$. From Equation (6–13),

$$a = \frac{[0.86 \times 620]^2}{310} = 917.10 \text{ MPa}$$

From Equation (6–14),

$$b = -\frac{1}{3}\log\left[\frac{0.86 \times 620}{310}\right] = -0.0785$$

Thus, Equation (6–12) is

$$S_f = 917.10\, N^{-0.0785}$$

$$S_f = 917\,(10^4)^{-0.0785} = 445\ \mathrm{MPa}$$

$$S_f = a N^b \Rightarrow N^b = \frac{S_f}{a} \Rightarrow N = \left(\frac{S_f}{a}\right)^{\frac{1}{b}}$$

$$S_f \rightarrow \sigma_a \quad \Rightarrow N = \left(\frac{\sigma_a}{a}\right)^{\frac{1}{b}}$$

$$\Rightarrow N = \left(\frac{379}{917}\right)^{\frac{1}{-0.0785}} = 77318$$

$$= 7.7 \times 10^4$$ سیکل

$\sigma_a = 310 \longrightarrow N \simeq 10^6$

$\sigma_a < 310\,MPa \longrightarrow N \rightarrow \infty$

$\sigma_a \simeq 379\,MPa \longrightarrow N \simeq 77000$

در تنش‌های استاتیکی، که با S_y و S_{ut} سروکار داریم

همان S_y و S_{ut} بدست آمده از آزمایش کشش ساده

را برای قطعات مختلف در شرایط مختلف بکار

می‌بریم

اما برای حد دوام این گونه نیست.

در تنش‌های نوسانی و خستگی

ضرائب اصلاح حد دوام

$$S_e = K_a K_b K_c K_d K_e S'_e$$

$$S_e = k_a k_b k_c k_d k_e S'_e$$

k_a = surface factor

k_b = size factor

k_c = load factor

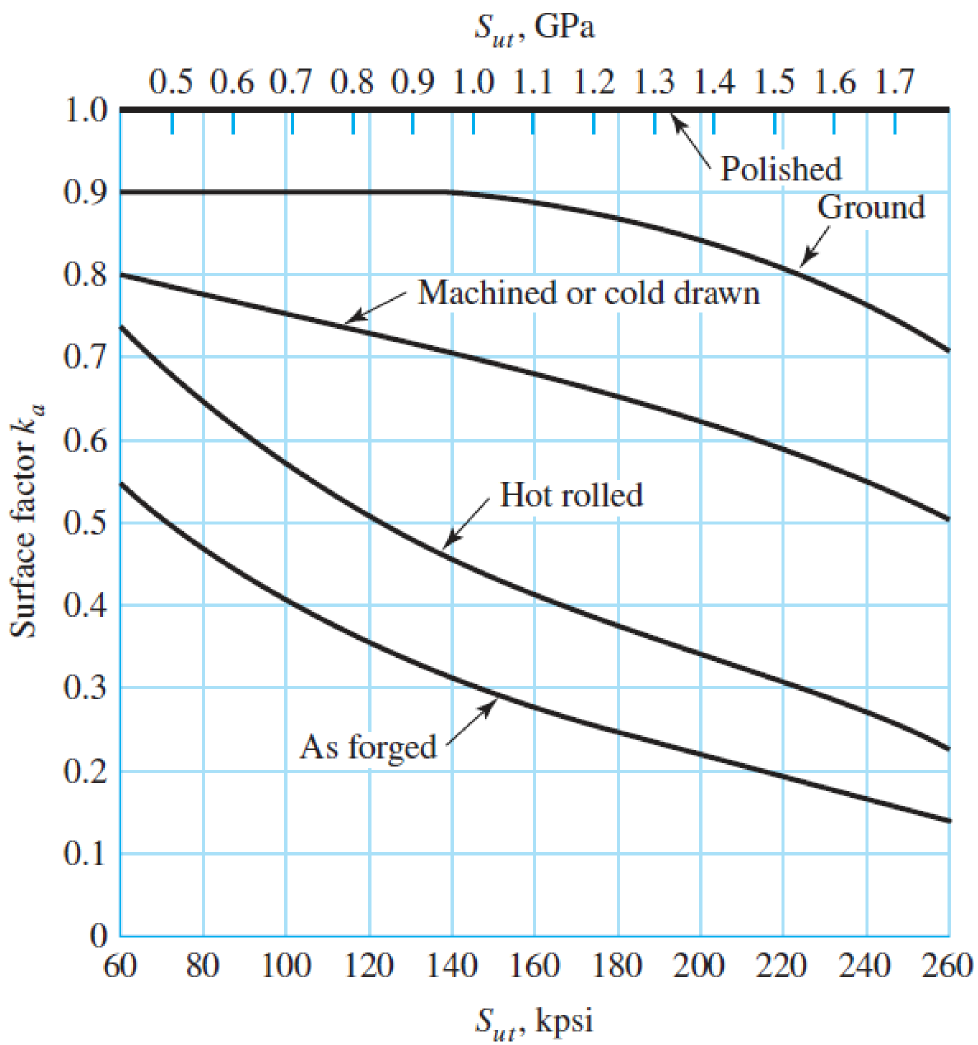
k_d = temperature factor

k_e = reliability factor

S'_e = rotary-beam test specimen endurance limit $= 0.5\, S_{ut}$

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

ضریب اصلاح سطح K_a



$$K_a = a\, S_{ut}^{\,b}$$

$$K_a = a\, S_{ut}^{\,b}$$

Table 6–2 **Curve Fit Parameters for Surface Factor, Equation (6–18)**

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758

EXAMPLE 6–3

A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate k_a.

Solution

From Table 6–2, $a = 3.04$ and $b = -0.217$. Then, from Equation (6–18)

Answer

$$k_a = 3.04(520)^{-0.217} = 0.78$$

در مسائل ما اغلب $K_a = 0.7$ تا 0.8

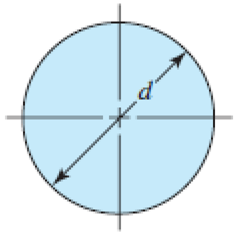
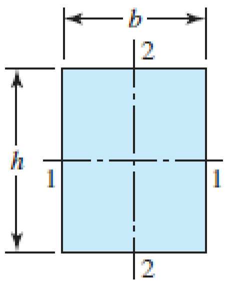
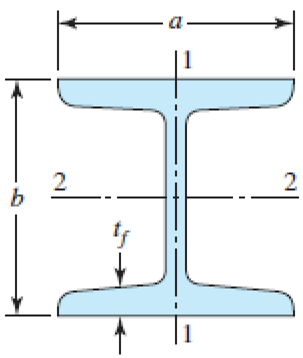
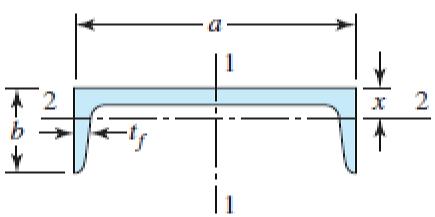
K_b ضریب اصلاح اندازه

برای بارگذاری خمشی و پیچشی برای تیرهای دوار

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.3 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 7.62 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases}$$

برای بارگذاری محوری $K_b = 1$

برای تیرهای غیر دوار

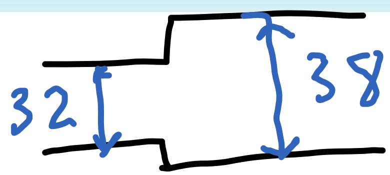
	$A_{0.95\sigma} = 0.01046d^2$ $d_e = 0.370d$
	$A_{0.95\sigma} = 0.05hb$ $d_e = 0.808\sqrt{hb}$
	$A_{0.95\sigma} = \begin{cases} 0.10at_f & \\ 0.05ba & t_f > 0.025a \end{cases}$ axis 1-1, axis 2-2
	$A_{0.95\sigma} = \begin{cases} 0.05ab \\ 0.052xa + 0.1t_f(b-x) \end{cases}$ axis 1-1, axis 2-2

EXAMPLE 6–4

A steel shaft loaded in bending is 32 mm in diameter, abutting a filleted shoulder 38 mm in diameter. Estimate the Marin size factor k_b if the shaft is used in

(*a*) A rotating mode.

(*b*) A nonrotating mode.



$d = 32$

Solution

(*a*) From Equation (6–19)

Answer

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{32}{7.62}\right)^{-0.107} = 0.86$$

(*b*) From Table 6–3,

$$d_e = 0.37d = 0.37(32) = 11.84 \text{ mm}$$

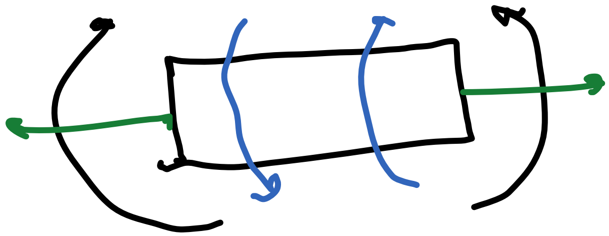
From Equation (6–19),

Answer

$$k_b = \left(\frac{11.84}{7.62}\right)^{-0.107} = 0.95$$

در طراحی k_b را عددی حدود 0.8 می‌توان فرض کرد

ضریب بار K_c



$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

اغلب موارد بارگذاری از یک نوع
نیست و بارگذاری ترکیبی است
دقیقی هم تحت وهم پیچش و خمش
است مجبوریم هم واسنجی کنیم.

بنابراین ما $K_c=1$ می‌گیریم و تأثیر نوع بار را در محاسبه تنش‌ها
اعمال می‌کنیم

ضریب اصلاح دما K_d

برای $T<350^\circ C$ $\longleftarrow$ $K_d \simeq 1$

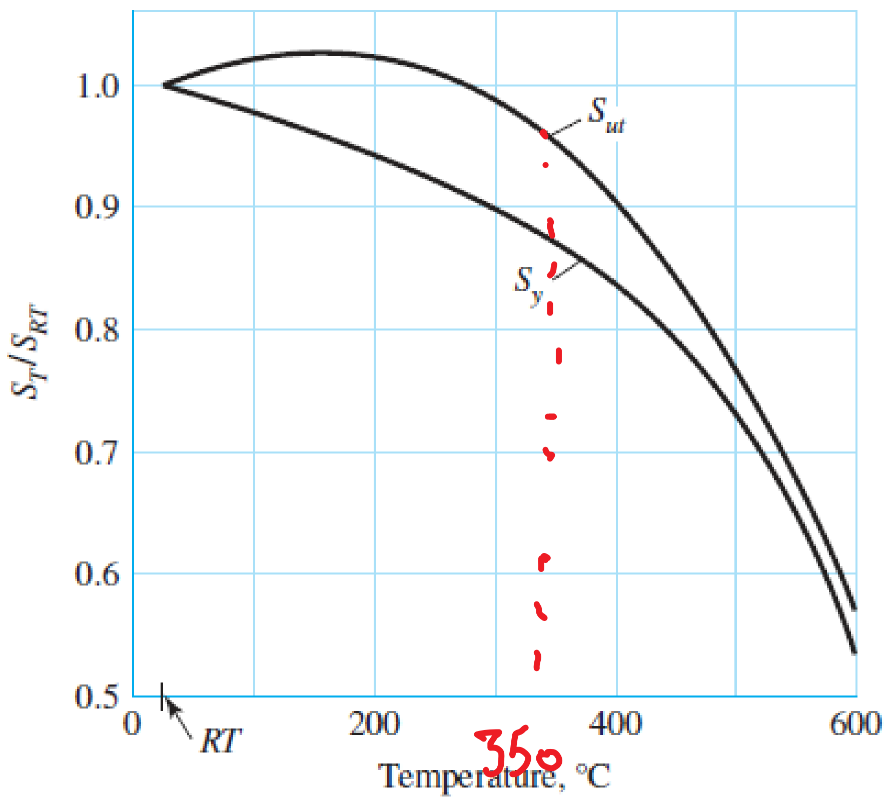
با خنک کاری مناسب بالا رفتن دمای سطوح

اغلب دمای کاری ماشین‌آلات زیر $50^\circ C$ باشد.

برای مسئله حل شده کتاب: $T \simeq 400^\circ C \rightarrow K_d = 0.9$

فولاد استیل

شکل 2-17



$$\frac{(S_{ut})_T}{(S_{ut})_{RT}} = \frac{S_T}{S_{RT}}$$

دمای اتاق

$$S_T/S_{RT} = 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2$$

$$S_T/S_{RT} = 0.99 + 5.9(10^{-4})T_C - 2.1(10^{-6})T_C^2$$

$$\frac{(S_{ut})_{400}}{(S_{ut})_{24}} = 0.9$$

$$T < 350 \rightarrow \frac{S_T}{S_{RT}} \simeq 1$$

حالت اول S_{ut} در دمای اتاق داده شود:

$$(S_{ut})_T = \frac{S_T}{S_{RT}} (S_{ut})_{24^0}$$

$$(S'_e)_T = 0.5 \, (S_{ut})_T$$

حالت دوم S'_e در دمای اتاق

$$(S'_e)_T = \overbrace{\frac{S_T}{S_{RT}}}^{K_d} (S'_e)_{24} = K_d (S'_e)_{24^0}$$

$$(S_{uT})_{21^\circ C} = 551.5\,\text{MPa}\ ,\ (S_{ut})_{398} = 0.89(S_{ut})_{21^\circ}$$

EXAMPLE 6–5

A 1035 steel has a tensile strength of 551.58 MPa and is to be used for a part that operates in a steady temperature of 398.89 C. Estimate the endurance limit at the operating temperature if
(*a*) only the tensile strength at room temperature is known.
(*b*) the room-temperature endurance limit for the material is found by test to be $(S'_e)_{21^\circ} = 268.89$ MPa.

Solution

(*a*) Estimate the tensile strength at the operating temperature from Equation (6–26),

$$(S_T/S_{RT})_{398.89^\circ} = 0.99 + 5.9(10^{-4})(398.89) - 2.1(10^{-6})(398.89)^2 = 0.89$$

Thus, $$(S_{ut})_{398.89^\circ} = (S_T/S_{RT})_{750^\circ}(S_{ut})_{70^\circ} = 0.89(551.58) = 491.57\text{ MPa}$$

21

From Equation (6–10),

Answer $$(S_e)_{398.89^\circ} = 0.5(S_{ut})_{398.89^\circ} = 0.5(491.57) = 245.78\text{ MPa}$$

and use $k_d = 1$ since this is already adjusted for the operating temperature.
(*b*) Since the endurance limit is known at room temperature, apply the temperature factor to adjust it to the operating temperature. From Equation (6–27),

$$k_d = (S_T/S_{RT})_{398.89^\circ} = 0.89$$

Answer $$(S_e)_{398.89^\circ} = k_d(S'_e)_{21^\circ} = 0.89(268.89) = 239.64\text{ MPa}$$

k_e ضریب قابلیت اطمینان

Table 6–4 Reliability Factors k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702

دکتر امین نیکوبین

منظور از قابلیت اطمینان ۹۹٪

از بین ۱۰۰ نمونه فقط ۱ نمونه دچار شکست شود.

۹۹.۹٪ از بین ۱۰۰۰ نمونه فقط یک عدد خراب باشد

۹۹.۹۹٪ 〃 ۱۰۰۰۰ 〃 〃 〃 〃

۵۰٪ از ۱۰۰ نمونه ۵۰ سالم ۵۰ ناسالم

$$S_e = \underbrace{K_a \, K_b \, K_c \, K_d \, K_e}_{1} \, S'_e$$

$K_c = K_d = K_e = 1$ در حل مسائل اغلب

$$S_e = K_a \, K_b \, S'_e$$

مقدار تقریبی برای K_a و K_b ، 0.8

$$S_e = 0.8 \times 0.8 \times 0.5 \, S_{ut} = 0.32 \, S_{ut}$$

EXAMPLE 6–6

A 1080 hot-rolled steel bar has been machined to a diameter of 25.4 mm. It is to be placed in reversed axial loading for 70 000 cycles to failure in an operating environment of 343.33 °C. Using ASTM minimum properties, and a reliability for the endurance limit estimate of 99 percent, estimate the endurance limit and fatigue strength at 70 000 cycles.

Solution

From Table A–20, $S_{ut} = 772.21$ MPa at 21.11 C. Since the rotating-beam specimen endurance limit is not known at room temperature, we determine the ultimate strength at the elevated temperature first, using Equation (6–26),

$$(S_T/S_{RT})_{650^\circ} = 0.99 + 5.9(10^{-4}) \times 343.33 - 2.1(10^{-6}) \times 343.33^2 = 0.9450$$

343

The ultimate strength at 343.33 °C is then

$$(S_{ut})_{343.33^\circ} = (S_T/S_{RT})_{343.33^\circ}(S_{ut})_{21.11^\circ} = 0.945(772.21) = 729.75 \text{ MPa}$$

The rotating-beam specimen endurance limit at 343.33 °C is then estimated from Equation (6–10) as

$$S_e' = 0.5(729.75) = 364.88 \text{ MPa}$$

Next, we determine the Marin factors. For the machined surface, Equation (6–18) with Table 6–2 gives

$$k_a = aS_{ut}^b = 3.04(729.75)^{-(0.217)} = 0.73$$

For axial loading, from Equation (6–20), the size factor $k_b = 1$, and from Equation (6–25) the loading factor is $k_c = 0.85$. The temperature factor $k_d = 1$, since we accounted for the temperature in modifying the ultimate strength and consequently the endurance limit. For 99 percent reliability, from Table 6–4, $k_e = 0.814$. The endurance limit for the part is estimated by Equation (6–17) as

Answer

$$S_e = k_a k_b k_c k_d k_e S_e'$$
$$= 0.73(1)(0.85)(1)(0.814)364.88 = 183.55 \text{ MPa}$$

جلسه دوازدهم

تمرکز تنش و حساسیت به چاک

تنش ثابت

برای حالت بارگذاری ثابت و مواد ترد

$$\sigma_{max} = K_t \sigma_o$$

$$\tau_{max} = K_{ts} \tau_o$$

برای بارگذاری نوسانی و شکست خستگی (ترد و نرم)

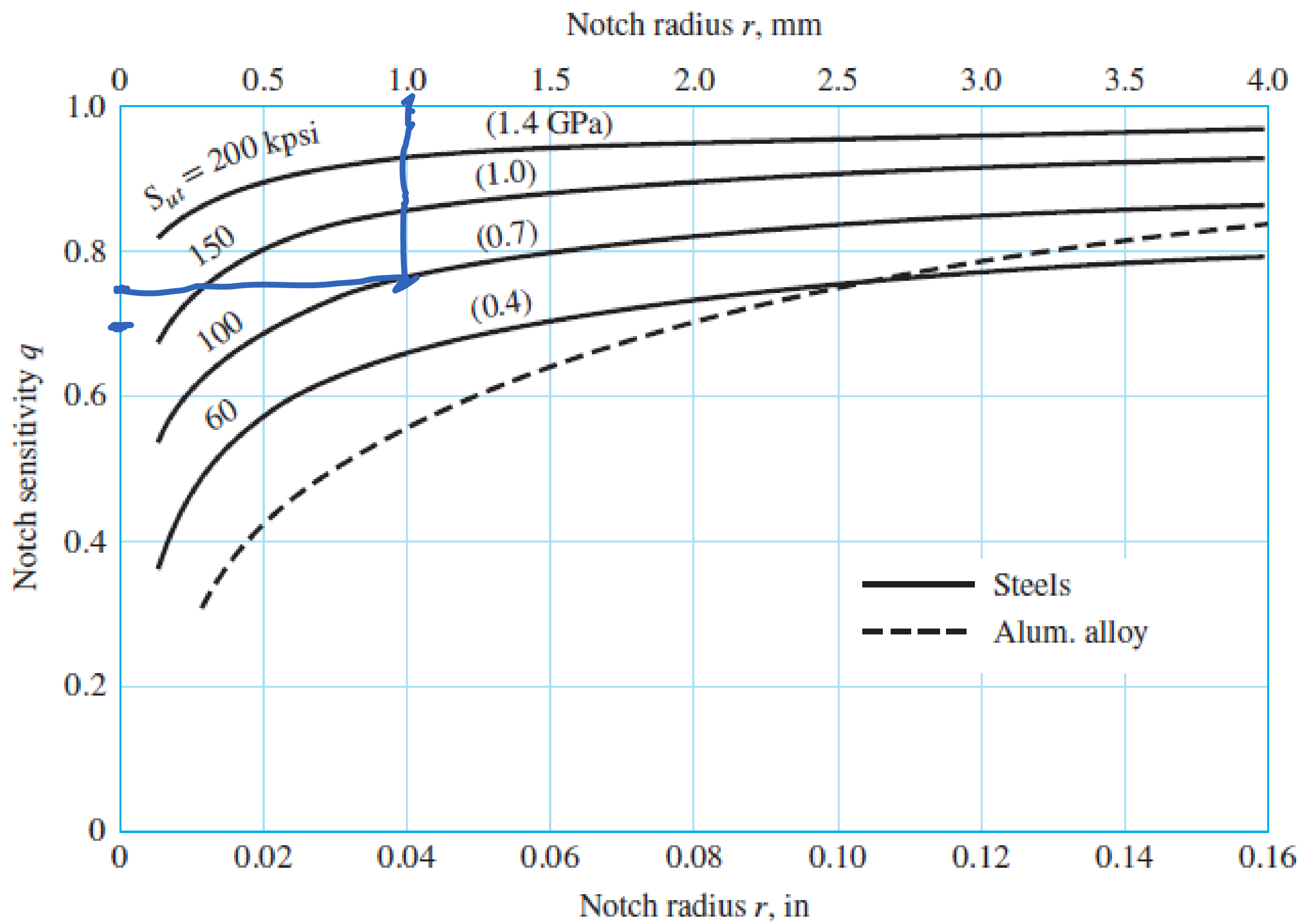
$$\sigma_{max} = K_f \sigma_o \quad , \quad K_f = 1 + q(K_t - 1)$$

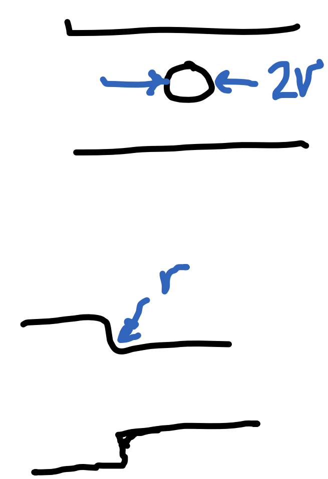
$$\tau_{max} = K_{fs} \tau_o \quad , \quad K_{fs} = 1 + q_s(K_{ts} - 1)$$

ضریب حساسیت به چاک q, q_s

شکل 6-26

(q)

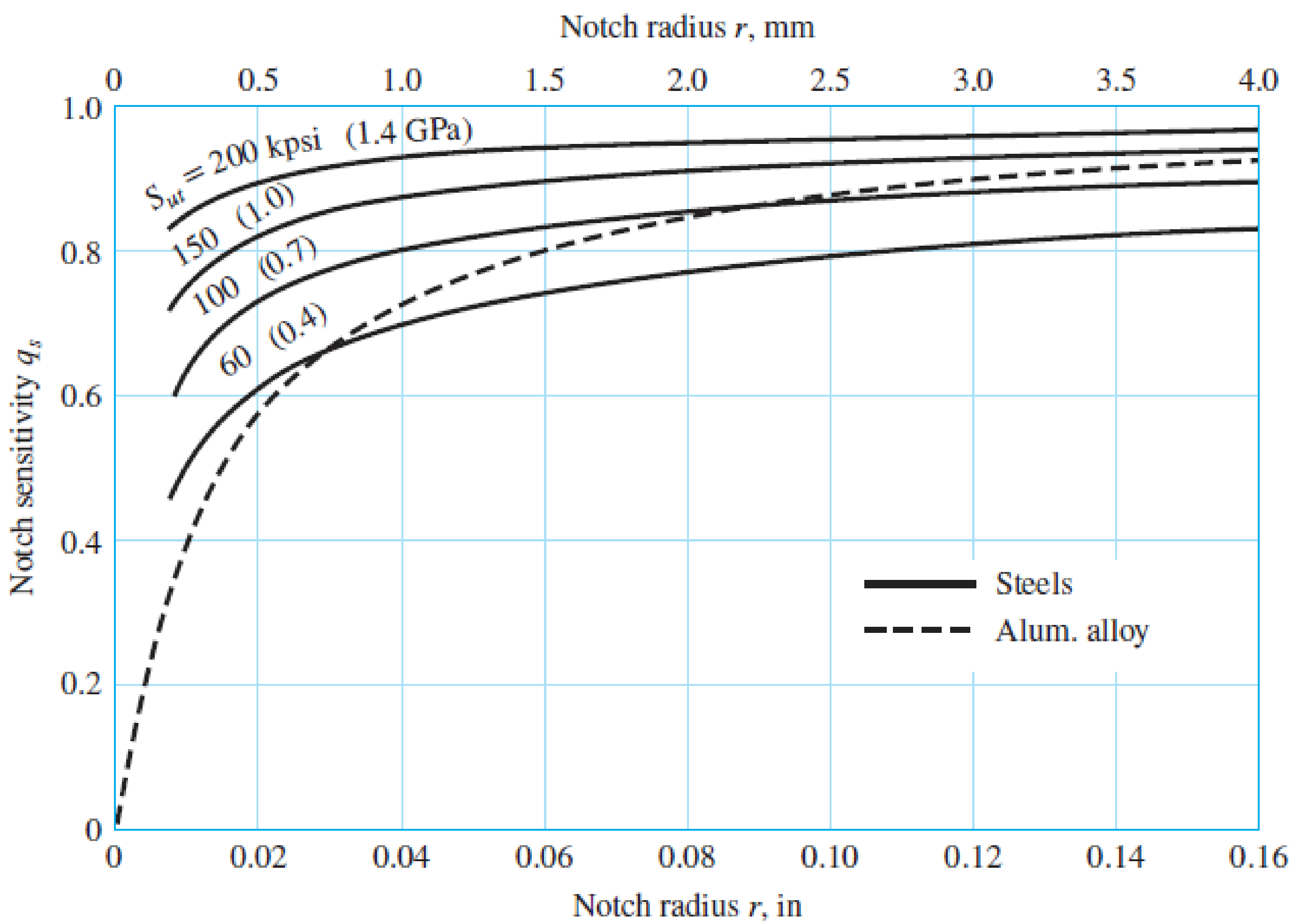




$q = 0.75 \Leftarrow r = 1\,mm$, $S_{ut} = 700\,Mpa = 0.7\,GPa$

شکل 6-27

q_s


Notch radius r, mm
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1.0
0.8
0.6
0.4
0.2
0
Notch sensitivity q_s
S_{ut} = 200 kpsi (1.4 GPa)
150 (1.0)
100 (0.7)
60 (0.4)
Steels
Alum. alloy
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
Notch radius r, in

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} \tag{6–33}$$

where r is the notch radius. For convenience, we can combine Equations (6–32) and (6–33), giving

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a}/\sqrt{r}} \tag{6–34}$$

The Neuber constant is experimentally determined for each material. For steels, it correlates with the ultimate strength, and can be represented with the following curve-fit equations in both U.S. customary and SI units.

Bending or axial:

q

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \qquad 50 \le S_{ut} \le 250 \text{ kpsi}$$
$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.60(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \qquad 340 \le S_{ut} \le 1700 \text{ MPa} \tag{6–35}$$

Torsion:

q_s

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \qquad 50 \le S_{ut} \le 220 \text{ kpsi}$$
$$\sqrt{a} = 0.958 - 1.83(10^{-3})S_{ut} + 1.43(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \qquad 340 \le S_{ut} \le 1500 \text{ MPa} \tag{6–36}$$

EXAMPLE 6–7

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using:
(a) Figure 6–26.
(b) Equations (6–34) and (6–35).

Solution

From Figure A–15–9, using $D/d = 38/32 = 1.1875$, $r/d = 3/32 = 0.093\,75$, we read the graph to find $K_t = 1.65$.
(a) From Figure 6–26, for $S_{ut} = 690$ MPa and $r = 3$ mm, $q = 0.84$. Thus, from Equation (6–32)

Answer

$$K_f = 1 + q(K_t - 1) = 1 + 0.84(1.65 - 1) = 1.55$$

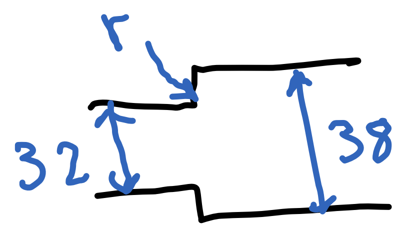
(b) From Equation (6–35) with $S_{ut} = 690$ MPa, $\sqrt{a} = 0.314\sqrt{\text{mm}}$. Substituting this into Equation (6–34) with $r = 3$ mm gives

Answer

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + \dfrac{0.314}{\sqrt{3}}} = 1.55$$

$S_{ut} = 690\ \text{MPa} = 0.69\ \text{GPa}$

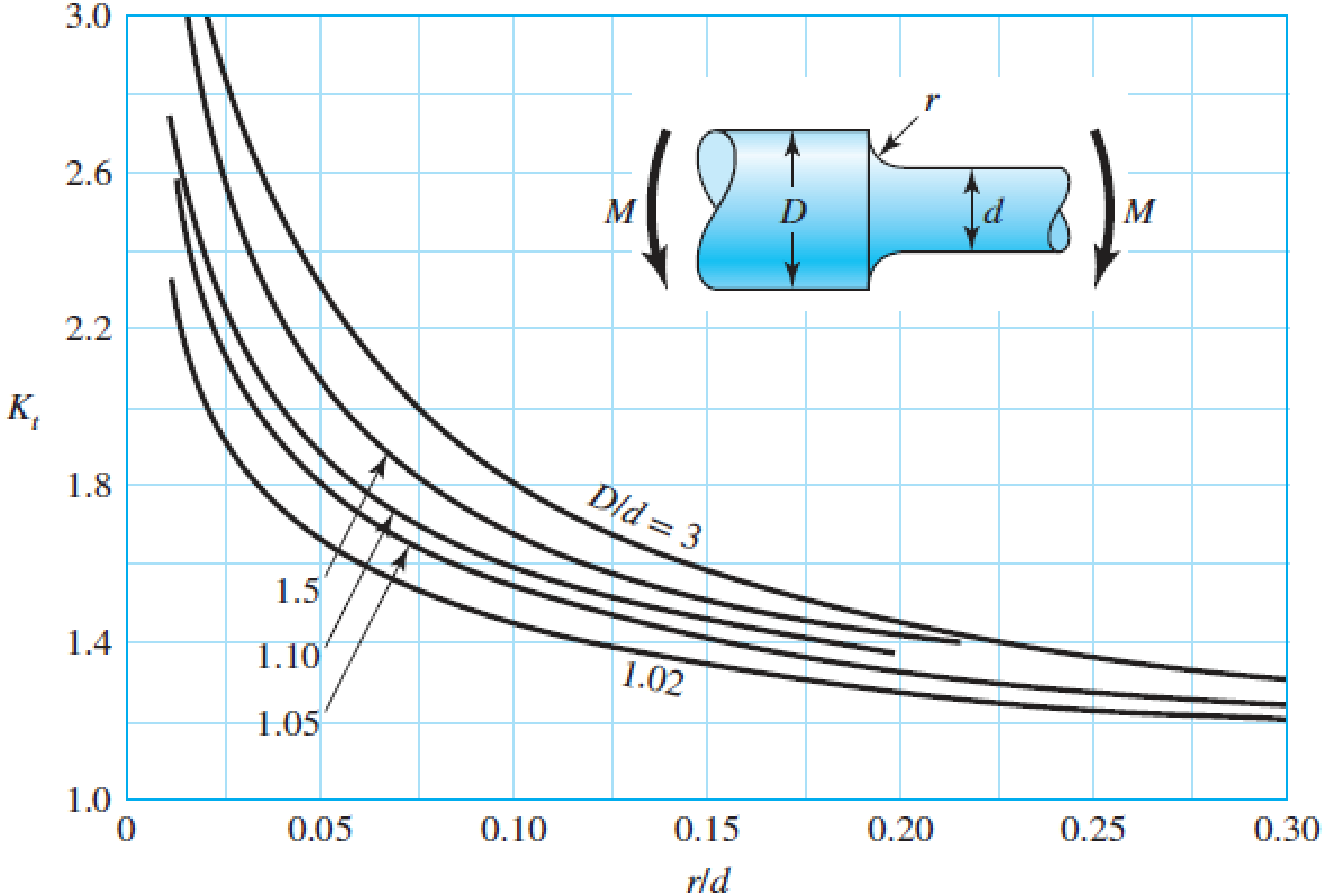
$r = 3\ \text{mm}$

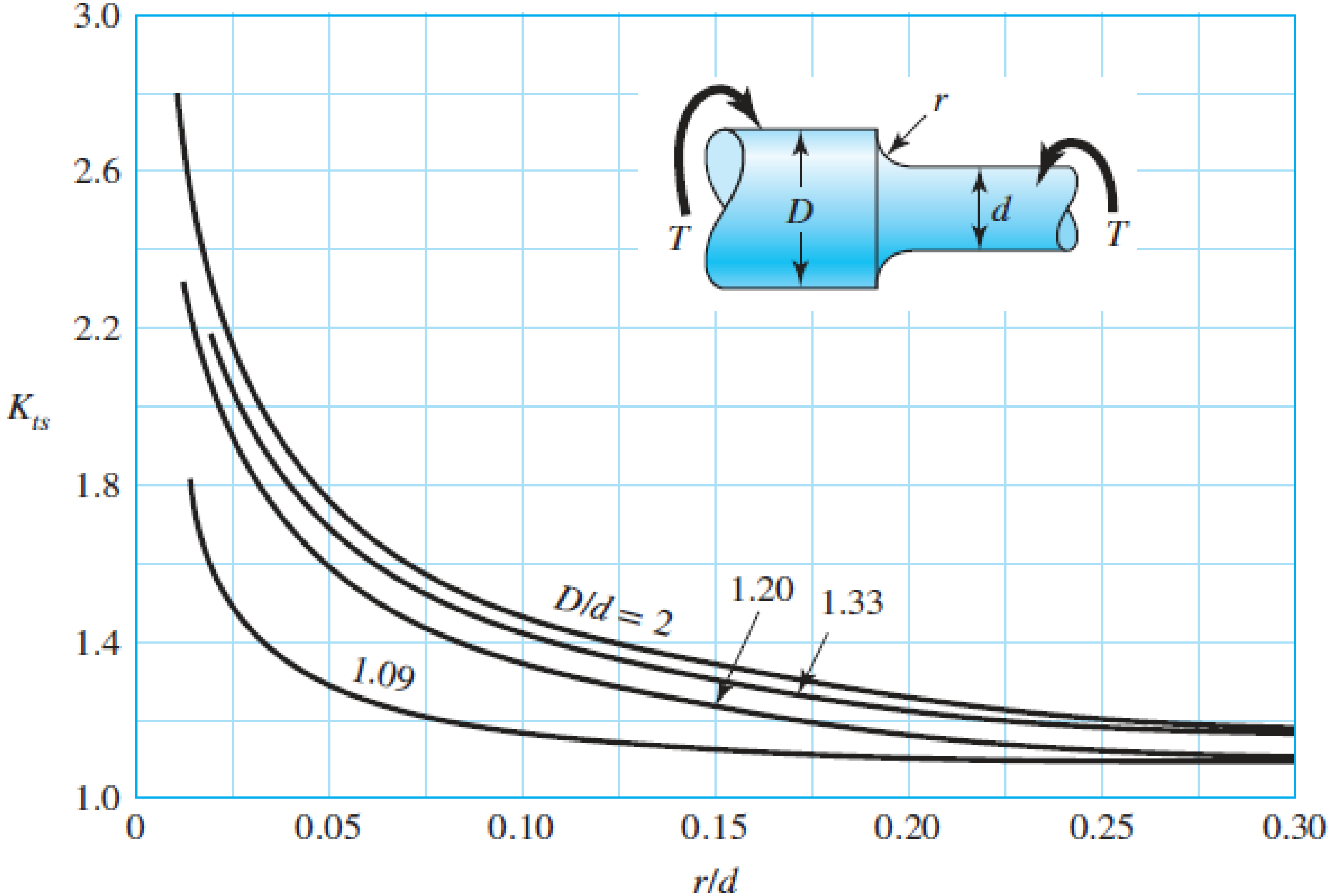


$\frac{D}{d} = 1.18$

$\frac{r}{d} = 3/32 = 0.09$

$K_t \simeq 1.6$


3.0
2.6
2.2
1.8
1.4
1.0
K_t
0
0.05
0.10
0.15
0.20
0.25
0.30
r/d
r
M
D
d
M
D/d = 3
1.5
1.10
1.05
1.02


3.0
2.6
2.2
1.8
1.4
1.0
K_{ts}
0
0.05
0.10
0.15
0.20
0.25
0.30
r/d
$D/d = 2$
1.20
1.33
1.09
r
T
D
d
T

جلسه سیزدهم

EXAMPLE 6–8

Figure 6–28*a* shows a rotating shaft simply supported in ball bearings at *A* and *D* and loaded by a nonrotating force *F* of 6.8 kN. The shaft is machined from AISI 1050 cold-drawn steel. Estimate the life of the part.

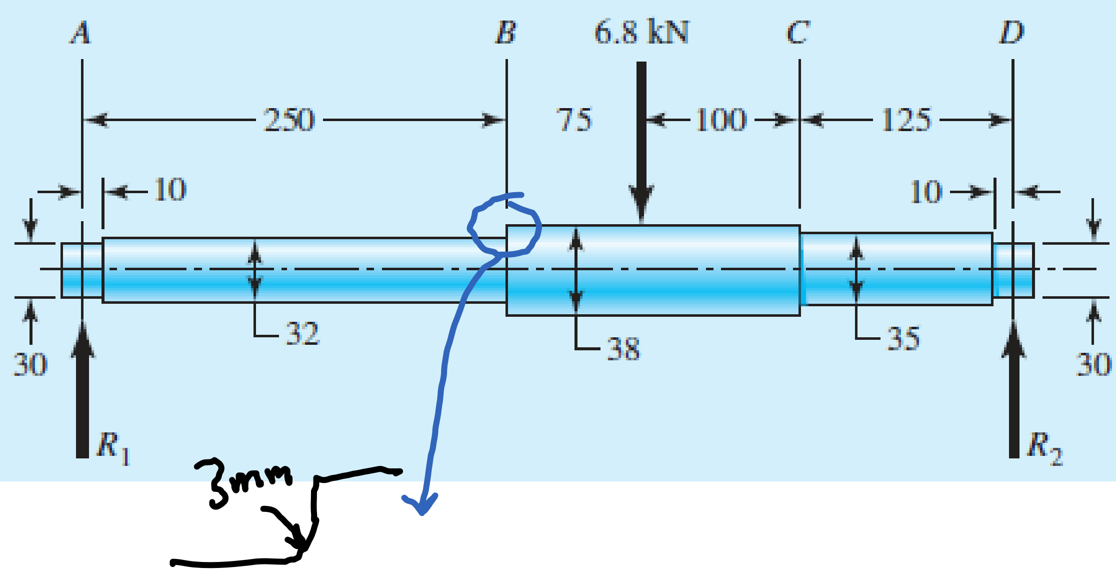


Figure 6–28

(*a*) Shaft drawing showing all dimensions in millimeters; all fillets 3-mm radius. (*b*) Bending-moment diagram.

$$N = \left(\frac{\sigma_a}{a}\right)^{\frac{1}{b}} \qquad a = \frac{(f S_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$$

$$\Rightarrow N = \left(\frac{335}{1564}\right)^{\frac{-1}{0.1415}} = 52000$$

سیکل

AISI1050

جدول $A-20 \Rightarrow S_{ut} = 690\ MPa$

$$S_e = K_a K_b\, S'_e = K_a K_b\,(0.5\, S_{ut})$$

$$K_c = K_d = K_e = 1$$

$$S'_e = 0.5(690) = 345 \text{ MPa}$$

From Equation (6–18) and Table 6–2,

$$k_a = 3.04(690)^{-0.217} = 0.74$$

From Equation (6–19),

$$k_b = (32/7.62)^{-0.107} = 0.86$$

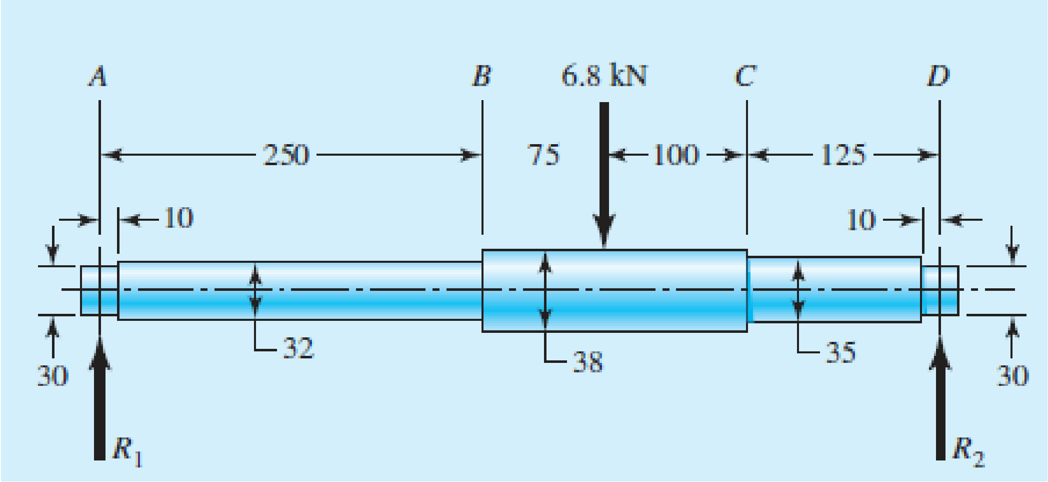
Since $k_c = k_d = k_e = 1$,

$$S_e = 0.74(0.86)345 = 220 \text{ MPa}$$

شکل ۶-۲۳ $\xRightarrow{S_{ut}=690}$ $f = 0.85$

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{[0.85(690)]^2}{220} = 1564 \text{ MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left[\frac{0.85(690)}{220}\right] = -0.1419$$

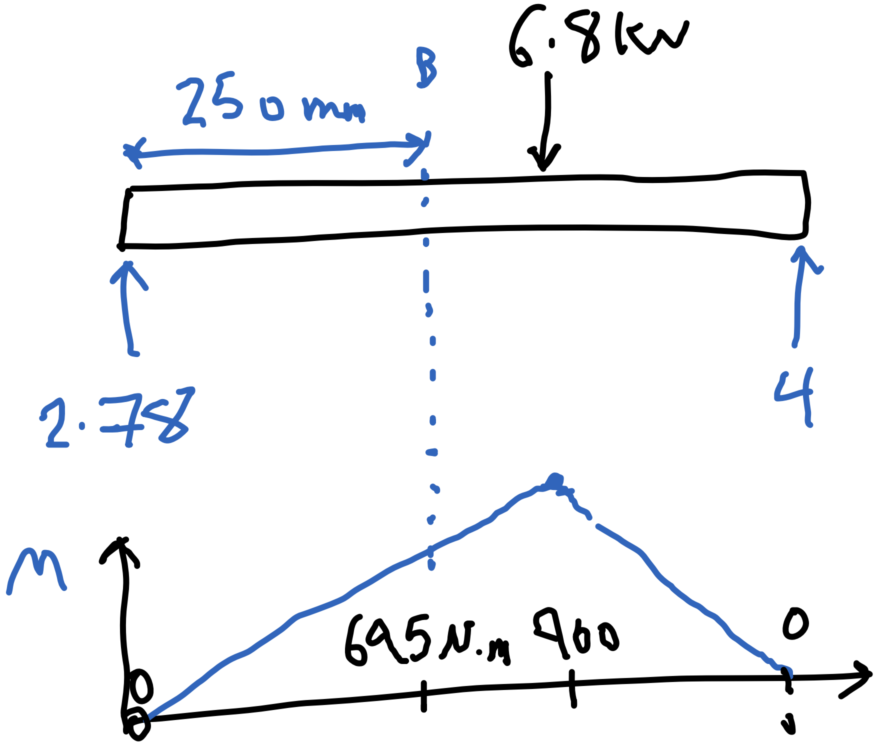


$$R_1 + R_2 = 6.8 \text{ kN}$$

$$\sum M_D = 0 \Rightarrow R_1 \times 550 - 6.8 \times 225 = 0$$

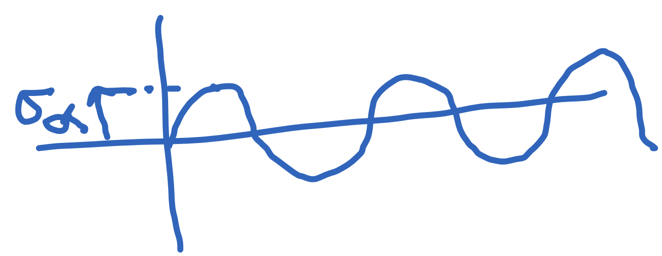
$$\Rightarrow R_1 = \frac{6.8 \times 225}{550} = 2.78 \text{ kN}$$

$$R_2 = 4 \text{ kN}$$



$$\sigma_{a0} = \frac{Mc}{I} = \frac{M\, d/2}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32 \times 695}{\pi (32 \times 10^{-3})^3}$$

$$\Rightarrow \quad \sigma_{a0} = 216\ \text{MPa}$$



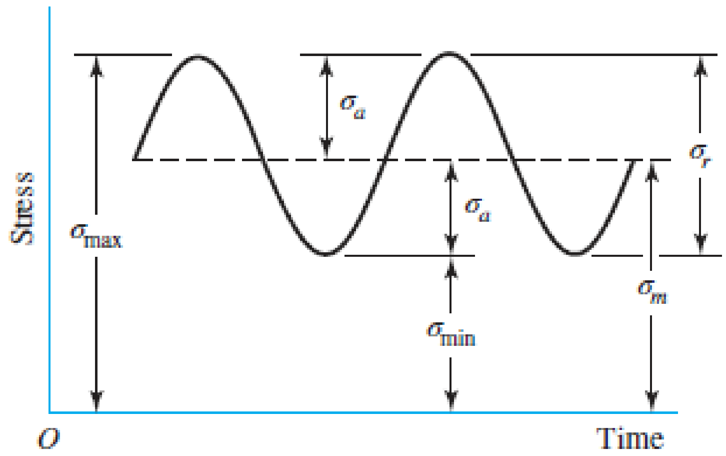
$$\sigma_a = K_f \, \sigma_{a0} = 1.55 \times 216 = 335 \text{ MPa}$$

$$K_f = 1 + q(K_t - 1)$$

$$K_t \Rightarrow \left\{ \begin{array}{l} D/d = \frac{38}{32} = 1.18 \\ r/d = \frac{3}{32} = 0.09 \simeq 0.1 \end{array} \right\} \Rightarrow K_t = 1.65$$

شکل 6-26 $q = 0.85 \Rightarrow K_f = 1 + 0.85(0.65) = 1.55$

معیارهای شکست خستگی



$\sigma_{\min}$ = minimum stress

$\sigma_{\max}$ = maximum stress

σ_a = alternating stress, or stress amplitude

σ_m = mean stress, or midrange stress

σ_r = stress range

The following relations are evident from Figure 6–29*d*. These were defined in Section 6–7 and are repeated here for convenience:

$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| \tag{6–8}$$

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} \tag{6–9}$$

In addition, the *stress ratio* is defined as

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{6–37}$$

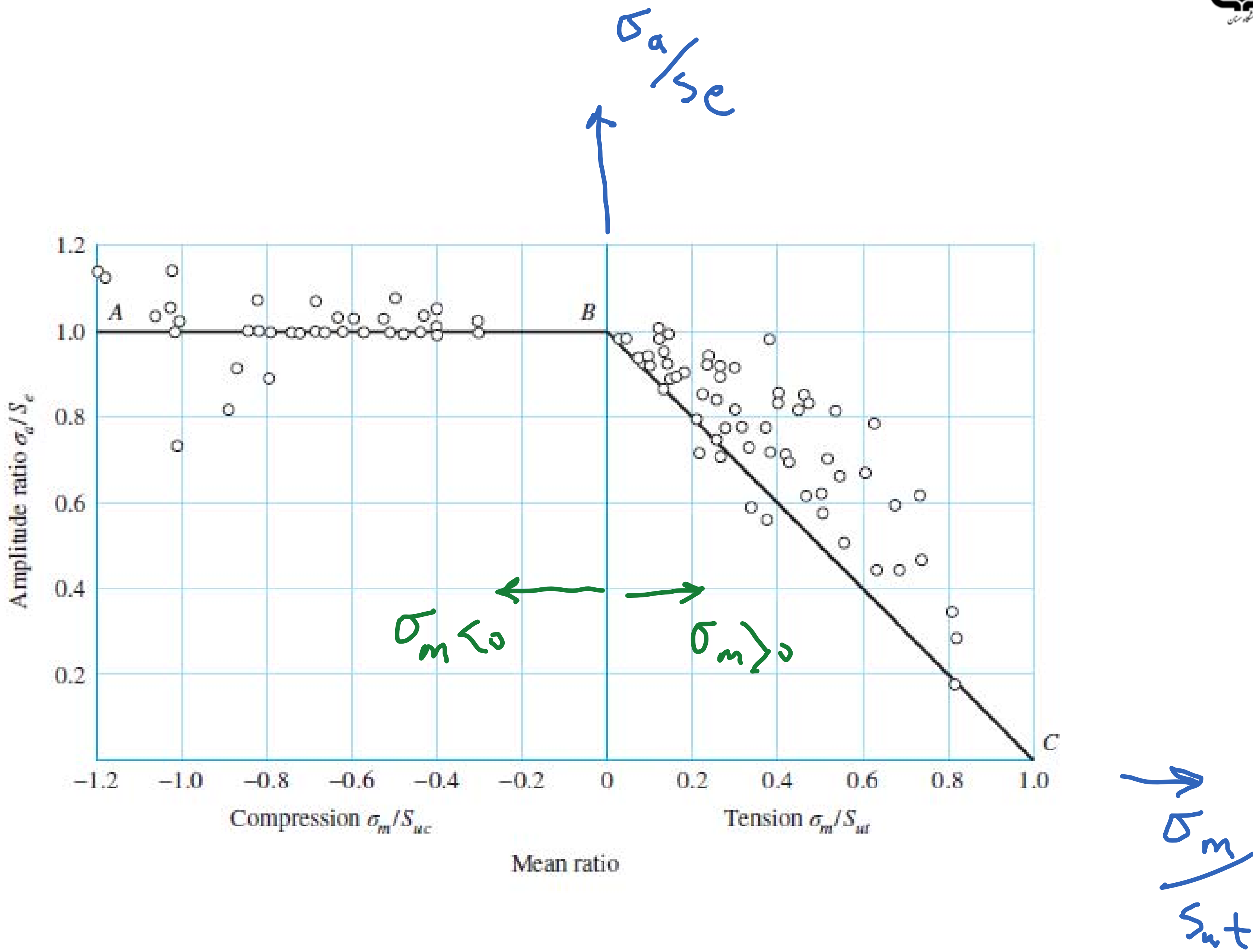
اعمال ضریب تمرکز تنش خستگی

هم در σ_a اعمال می‌شود.

هم در σ_m 〃 〃 .

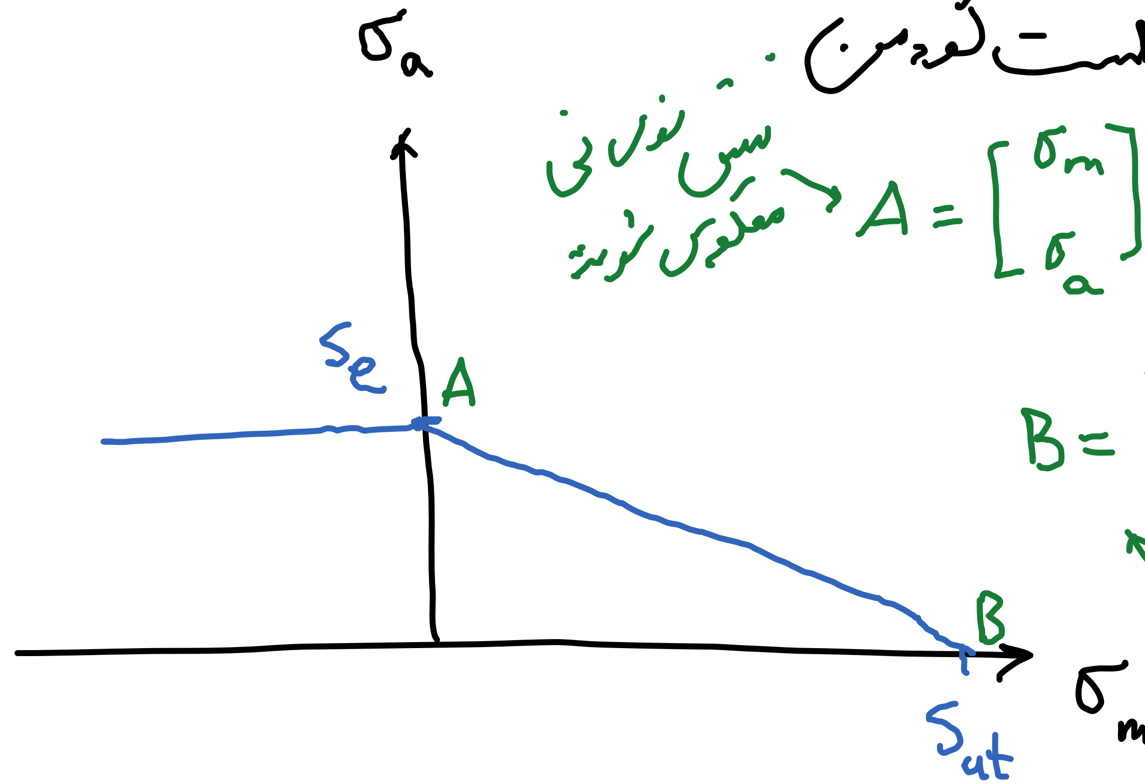
$$\sigma_a = K_f \, \sigma_{a0}$$

$$\sigma_m = K_f \, \sigma_{m0}$$



σ_a / S_e
1.2
1.0
0.8
0.6
0.4
0.2
A
B
C
Amplitude ratio σ_a / S_e
$\sigma_m < 0$
$\sigma_m > 0$
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1.0
Compression σ_m / S_{uc}
Tension σ_m / S_{ut}
Mean ratio
σ_m / S_{ut}

معیار شکست گودمن:



تنش نوسانی معکوس شونده

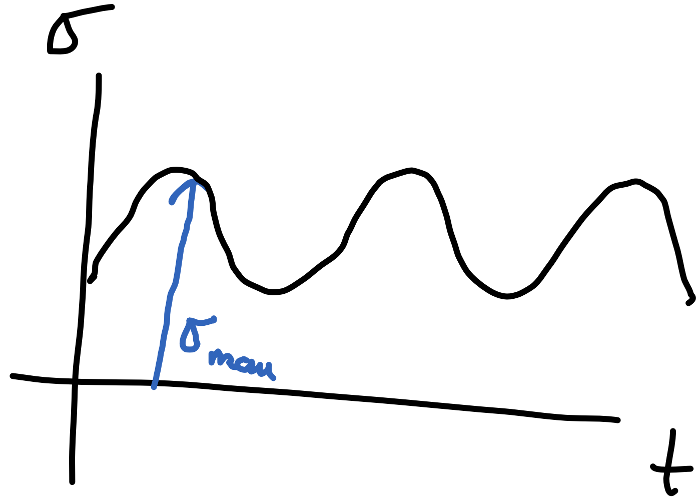
$$A = \begin{bmatrix} \sigma_m \\ \sigma_a \end{bmatrix} = \begin{bmatrix} 0 \\ S_e \end{bmatrix}$$

$$B = \begin{bmatrix} S_{ut} \\ 0 \end{bmatrix}$$

تنش ثابت

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = \frac{1}{n_f} \qquad \sigma_m > 0$$

$$\frac{\sigma_a}{S_e} = \frac{1}{n_f} \qquad \sigma_m < 0$$



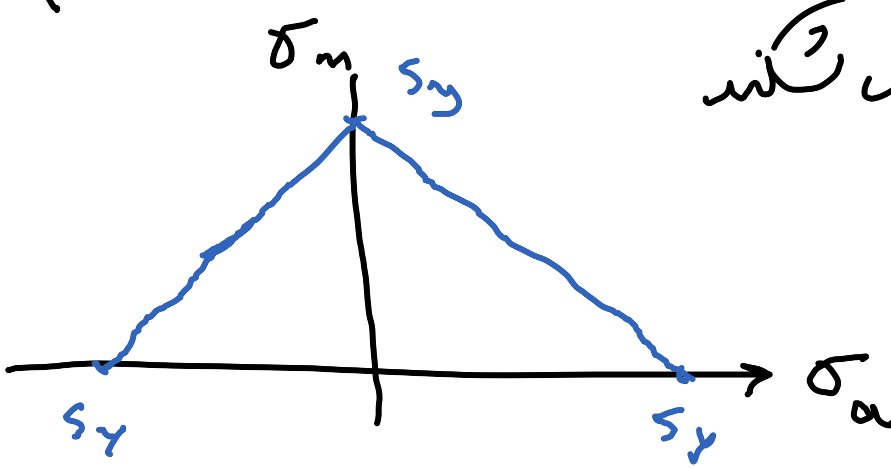
$$n_y = \frac{S_y}{\sigma_{max}} = \frac{S_y}{\sigma_m + \sigma_a}$$

ضریب اطمینان تسلیم استاتیکی

اگر تنش اعمالی بیشتر از S_y باشد $\sigma_{max} > S_y$

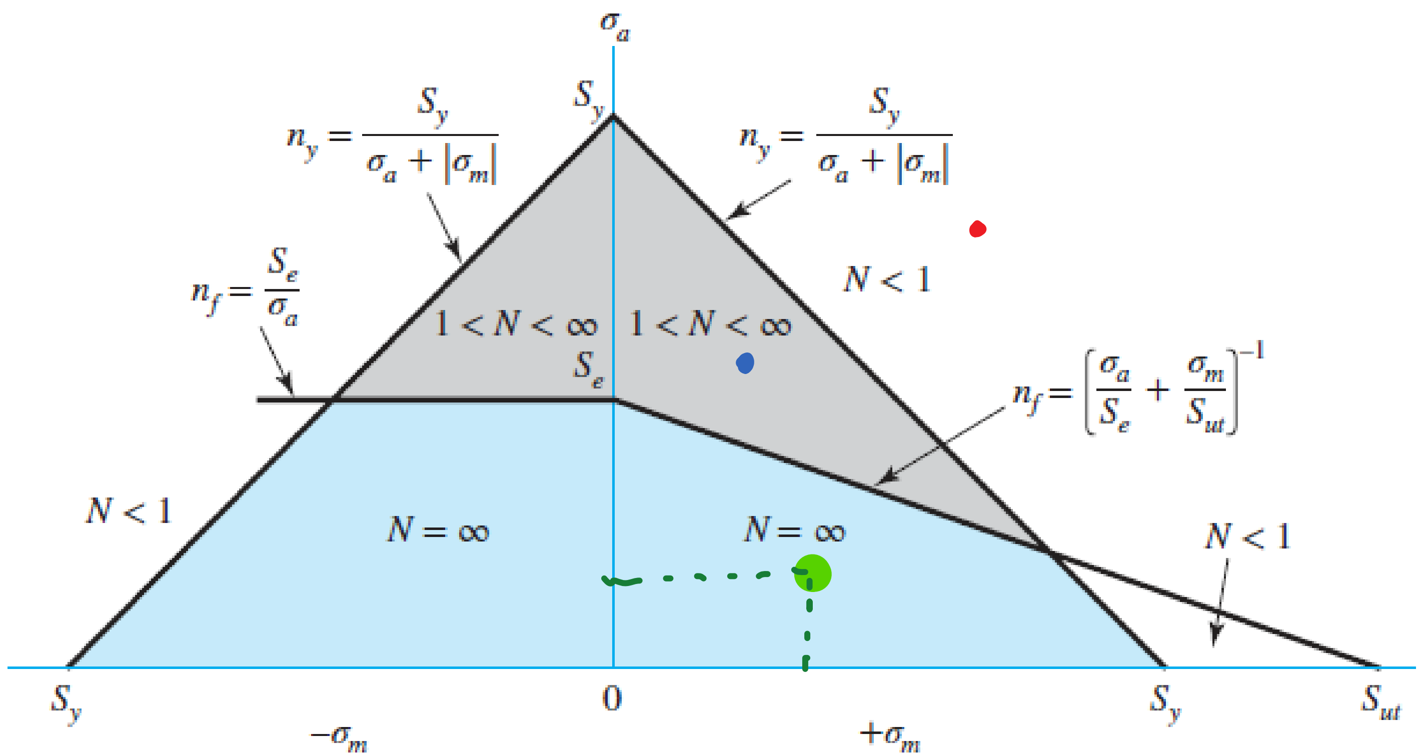
در این حالت عمر قطعه $N < 1$ و $n_y < 1$

در همان دور اول قطعه می شکند



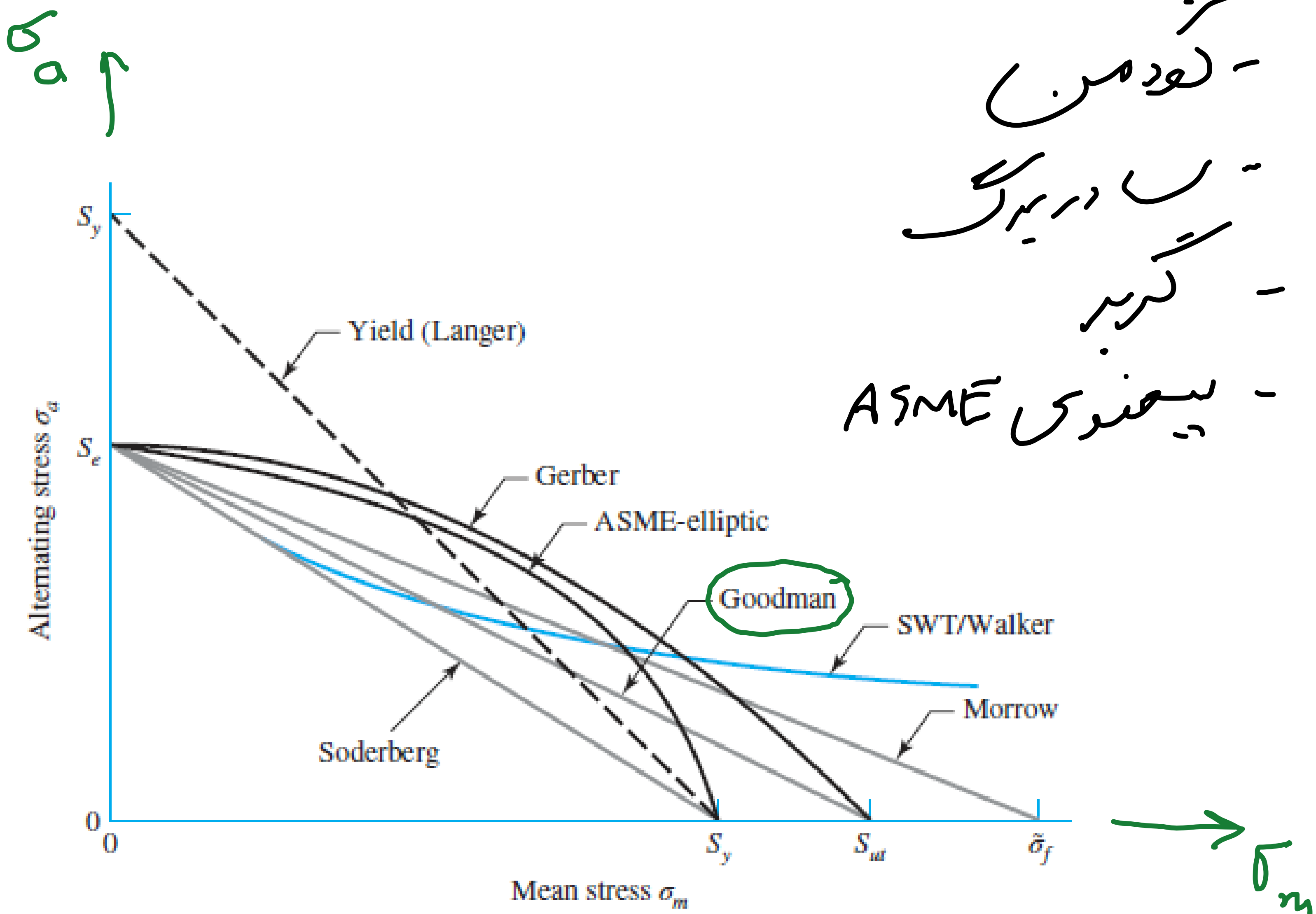
$n_y = 1 \Rightarrow \sigma_m + \sigma_a = S_y$

$\Rightarrow \sigma_m = S_y - \sigma_a$


σ_a
S_y
$n_y = \dfrac{S_y}{\sigma_a + |\sigma_m|}$
$n_y = \dfrac{S_y}{\sigma_a + |\sigma_m|}$
$n_f = \dfrac{S_e}{\sigma_a}$
$1 < N < \infty$
$1 < N < \infty$
S_e
$N < 1$
$n_f = \left(\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}\right)^{-1}$
$N < 1$
$N = \infty$
$N = \infty$
$N < 1$
S_y
0
S_y
S_{ut}
$-\sigma_m$
$+\sigma_m$

معیارهای مختلف شکست خستگی

- گودمن
- سودربرگ
- گربر
- بیضوی ASME



معیار گودمن

$$\boxed{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_f}}$$

معیار سادربرگ

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n_f}$$

معیار گربر

$$\frac{n_f \sigma_a}{S_e} + \left(\frac{n_f \sigma_m}{S_{ut}}\right)^2 = 1$$

معیار بیضوی ASME

$$\left(\frac{n_f \sigma_a}{S_e}\right)^2 + \left(\frac{n_f \sigma_m}{S_y}\right)^2 = 1$$

معیار گودمن

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} \right)^{-1} \qquad \sigma_m > 0$$

معیار سادربرگ

$$n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} \right)^{-1} \qquad \sigma_m > 0$$

معیار گربر

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \left(\frac{\sigma_a}{S_e}\right) \left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2} \right] \qquad \sigma_m \geq 0$$

معیار بیضوی ASME

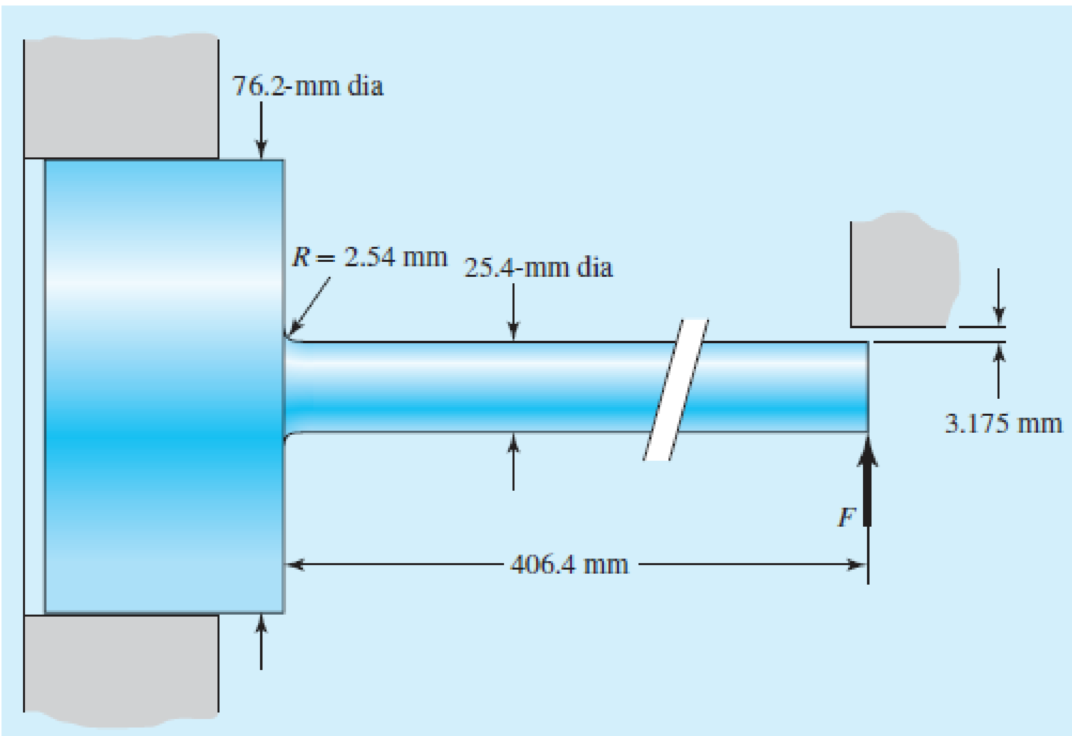
$$n_f = \left[\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2 \right]^{-1/2} \qquad \sigma_m \geq 0$$

برای همه معیارها

$$n_f = \frac{S_e}{\sigma_a} \qquad \sigma_m < 0$$

EXAMPLE 6–12

For the part shown in Figure 6–37, the 76-mm diameter end is firmly clamped. A force F is repeatedly applied to deflect the tip until it touches the rigid stop, then released. The part is machined from AISI 4130 quenched and tempered to a hardness of approximately 250 HB. Use Table A–23 for material properties. Estimate the fatigue factor of safety based on achieving infinite life, using each of the following criteria. Compare the results.
(*a*) Goodman (*b*) Morrow (*c*) Gerber



$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_f}$$

$$S_{ut} = 895 \text{ MPa}$$

$$S_e = k_a k_b (0.5 S_{ut})$$

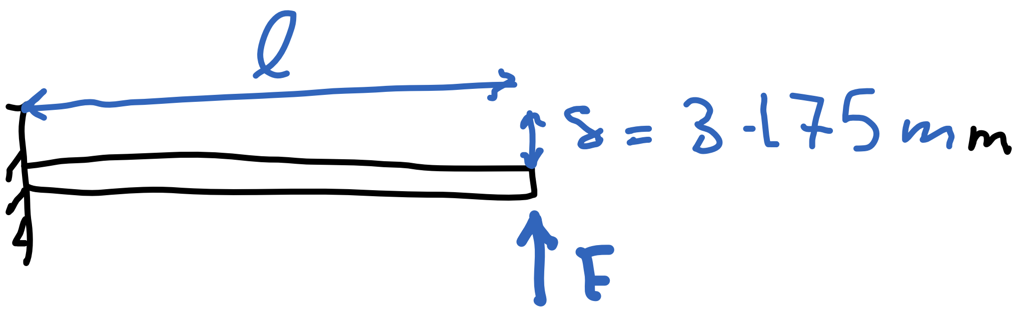
Equation (6–18): Machined $k_a = a(S_{ut})^b = 3.04(985)^{(-0.217)} = 0.70$

Equation (6–23): Nonrotating round $d_e = 0.37d = 0.37(25.4) = 9.398$

Equation (6–19): $k_b = 1.240d^{-0.107} = 1.240(0.37)^{(-0.107)} = 0.98$

Equations (6–10) and (6–17): $S_e = (0.70)(0.98)(0.5)(895) = 304$ MPa

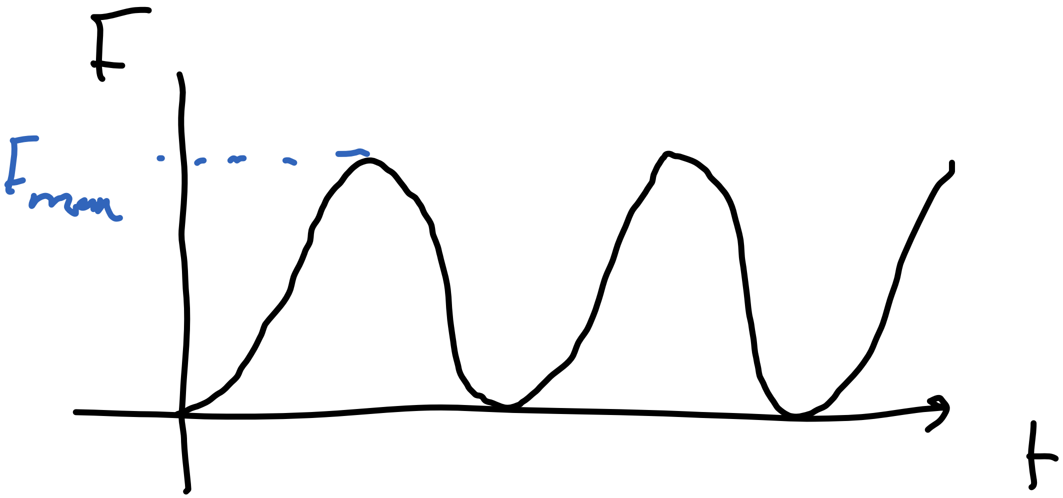
$E = 220$ GPa



$$I = \frac{\pi d^4}{64}$$

$$\delta = \frac{FL^3}{3EI} \Rightarrow F_{max} = \frac{3\delta_{max} EI}{L^3} = \frac{3 \times 3 \times 10^{-6}\, EI}{L^3}$$

$$\Rightarrow F_{max} = 638\,N$$



$$F_{min} = 0 \Rightarrow \sigma_{min} = 0$$

$$F_{max} = 638\ N \Rightarrow \sigma_{max_0} = \frac{MC}{I} = \frac{F_{max} \times l\ d/2}{\frac{\pi d^4}{64}} = \frac{32 F_{max} l}{\pi d^3} = \frac{32 \times 638 \times 406 \times 10^{-3}}{3.14 \times (25.4 \times 10^{-3})^3}$$

638x406×

$$\Rightarrow \sigma_{max_0} = 161.1\ MPa$$

$$\sigma_{max} = K_f \, \sigma_{max_0} = 1.7 \times 161.1 = 274 \, MPa$$

$$K_f = 1 + q_f (K_t - 1)$$

$$\left.\begin{aligned} r/d &= 0.1 \\ D/d &= 3 \end{aligned}\right\} \Rightarrow K_t = 1.8$$

$$\Rightarrow K_f = 1.7$$

$$q_f = 0.9$$

$$\sigma_{min} = 0 \quad , \quad \sigma_{max} = 274\ MPa$$

$$\sigma_m = \frac{\sigma_{min} + \sigma_{max}}{2} = \frac{274}{2} = 137\ MPa$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{274}{2} = 137\ MPa$$

معیار گودمن

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_f} \Rightarrow \frac{137}{304} + \frac{137}{895} = \frac{1}{n_f}$$

$$\Rightarrow n_f = 1.65$$

(*c*) Gerber

Equation (6–48):

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left(\frac{\sigma_a}{S_e}\right)\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

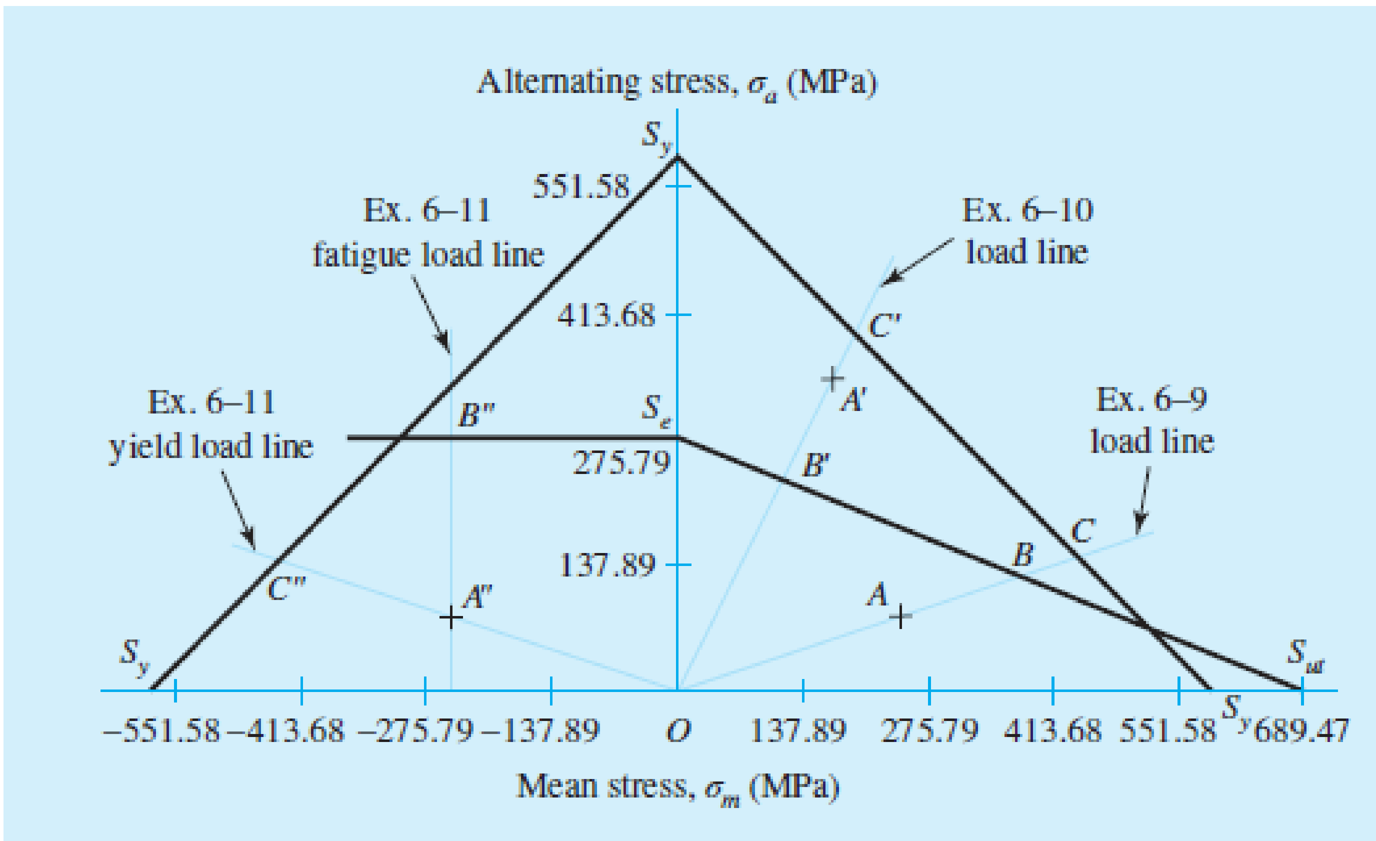
Answer

$$= \frac{1}{2}\left(\frac{895}{138.6}\right)^2\left(\frac{138.6}{303.7}\right)\left[-1+\sqrt{1+\left(\frac{2(138.6)(303.7)}{(895)(138.6)}\right)^2}\right] = 1.98$$

مثال‌های 6-9 ، 6-10 ، 6-11 ، 6-13

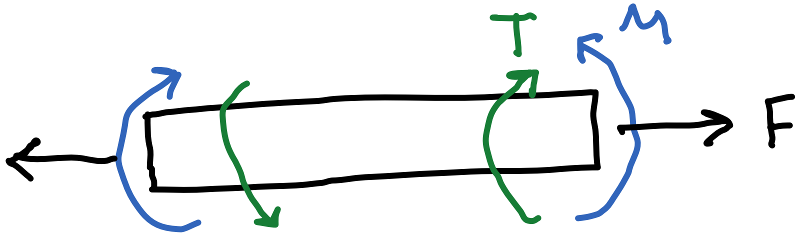
را بخوانید

شکل مربوط به مثالهای 6-9 ، 6-10 و 6-11



جلسه چهاردهم

بارگذاری ترکیبی



$F \Rightarrow F_{min} - F_{max}$

$M \rightarrow M_{min} - M_{max}$

$T \rightarrow T_{min} - T_{max}$

بار محوری F: $\sigma_{axial} = \dfrac{F}{A}$

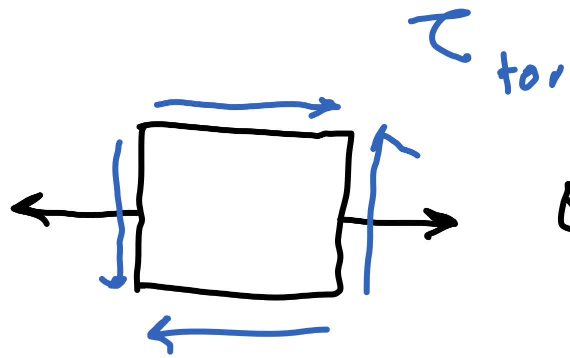
بار خمشی M: $\sigma_{bending} = \dfrac{Mc}{I}$

بار پیچشی T: $\tau_{tor} = \dfrac{Tc}{J}$

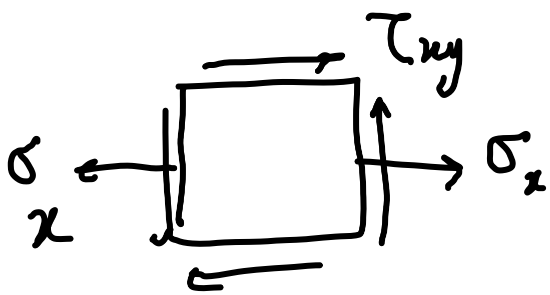
$\left.\begin{matrix}(\sigma_{min})_{axial} \\ (\sigma_{max})_{axial}\end{matrix}\right\} \rightarrow \left\{\begin{matrix}(\sigma_{a0})_{axial} \\ (\sigma_{m0})_{axial}\end{matrix}\right.$

$\left.\begin{matrix}(\sigma_{min})_{bending} \\ (\sigma_{max})_{bending}\end{matrix}\right\} \rightarrow \left\{\begin{matrix}(\sigma_{a0})_{bending} \\ (\sigma_{m0})_{bend}\end{matrix}\right.$

$\left.\begin{matrix}(\tau_{min})_{tor} \\ (\tau_{max})_{tor}\end{matrix}\right\} \rightarrow \left\{\begin{matrix}(\tau_{a0})_{torsion} \\ (\tau_{m0})_{tor}\end{matrix}\right.$



$$\sigma_{axial} + \sigma_{bending}$$



$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2}$$

$$\sigma'_m = \left\{ \left[(K_f)_{bending} (\sigma_{mo})_{bend} + (K_f)_{axial} (\sigma_{mo})_{axial} \right]^2 + 3\left[(K_{fs})_{tor} \times (\tau_{mo})_{tor} \right]^2 \right\}^{1/2}$$

$$\sigma'_a = \left\{ \left[(K_f)_{bending} (\sigma_{ao})_{bend} + \frac{(K_f)_{axial}}{0.85} (\sigma_{ao})_{axial} \right]^2 + 3\left[(K_{fs})_{tor} \times (\tau_{ao})_{tor} \right]^2 \right\}^{1/2}$$

رابطه گودمن

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{1}{n_f}$$

ضریب بار

$$(k_c)_{axial} = 0.85$$

$$(k_c)_{tors} = 0.57 \longrightarrow \frac{1}{\sqrt{3}}$$

$$(k_c)_{bend} = 1$$

جهت ساده سازی $(K_c)_{axial} = 1$ در نظر می‌گیرد.

$$\sigma'_a = \{[(K_f)_{\text{bending}}(\sigma_{a0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{a0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{a0})_{\text{torsion}}]^2\}^{1/2} \tag{6–66}$$

$$\sigma'_m = \{[(K_f)_{\text{bending}}(\sigma_{m0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{m0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{m0})_{\text{torsion}}]^2\}^{1/2} \tag{6–67}$$

اغلب بار محوری نداریم رابطه به صورت زیر ساده می‌شود.

$$\sigma'_a = \left\{ (K_f\, \sigma_{a0})^2_{bend} + 3(K_{fs}\, \tau_{a0})^2 \right\}^{1/2}$$

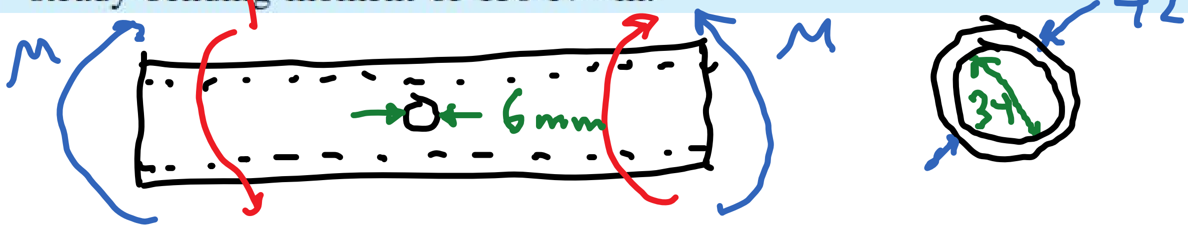
$$\sigma'_m = \left\{ (K_f\, \sigma_{m0})^2_{bend} + 3(K_{fs}\, \tau_{m0})^2 \right\}^{1/2}$$

EXAMPLE 6–17

A shaft is made of 42- × 4-mm AISI 1018 cold-drawn steel tubing and has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Goodman and Langer failure criteria for the following loading conditions:

(*a*) The shaft is rotating and is subjected to a completely reversed torque of 120 N · m in phase with a completely reversed bending moment of 150 N · m.

(*b*) The shaft is nonrotating and is subjected to a pulsating torque fluctuating from 20 to 160 N · m and a steady bending moment of 150 N · m.



a) $T_a = 120\ N.m\ ,\ T_m = 0$ شفت دوار

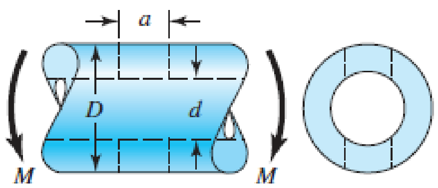
$M_a = 150\ ,\ M_m = 0$

b) $T_{min} = 20\ ,\ T_{max} = 160 \Rightarrow T_m = \frac{160+20}{2} = 90$

$T_a = \frac{160-20}{2} = 70$

$M_a = 0\ ,\ M_m = 150$

Table A–16 Approximate Stress-Concentration Factor K_t of a Round Bar or Tube with a Transverse Round Hole and Loaded in Bending



The nominal bending stress is $\sigma_0 = M/Z_{net}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

	d/D					
	0.9		0.6		0	
a/D	*A*	K_t	*A*	K_t	*A*	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

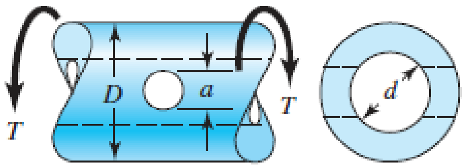
$D = 42 , \; d = 34 , \; a = 6$

$a/D = \frac{6}{42} = 0.14$

$d/D = \frac{34}{42} = 0.8$

$\Rightarrow A = 0.79 , \; K_t = 2.34$

Table A–16 Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion (*Continued*)



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{net}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{net} = \frac{\pi A(D^4 - d^4)}{32}$$

$$k_a = 3.04 S_{ut}^{-0.217} = 3.04(440)^{-0.217} = 0.81$$

tor is

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{42}{7.62}\right)^{-0.107} = 0.83$$

ll unity, so the modified endurance strength S_e is

$$S_e = 0.81(0.83)220 = 148 \text{ MPa}$$

(*a*) Theoretical stress-concentration factors are found from Table A–16. Using $a/D = 6/42 = 0.143$ and $d/D = 34/42 = 0.810$, and using linear interpolation, we obtain $A = 0.798$ and $K_t = 2.37$ for bending; and $A = 0.89$ and $K_{ts} = 1.75$ for torsion. Thus, for bending,

$$Z_{\text{net}} = \frac{\pi A}{32D}(D^4 - d^4) = \frac{\pi(0.798)}{32(42)}[(42)^4 - (34)^4] = 3.31(10^3)\,\text{mm}^3$$

and for torsion

$$J_{\text{net}} = \frac{\pi A}{32}(D^4 - d^4) = \frac{\pi(0.89)}{32}[(42)^4 - (34)^4] = 155(10^3)\,\text{mm}^4$$

$$K_f = 1 + q(K_t - 1) = 1 + 0.78(2.37 - 1) = 2.07$$

$$K_{fs} = 1 + 0.81(1.75 - 1) = 1.61$$

$$\begin{cases} M_a = 150 \\ T_a = 120 \end{cases}$$

$$\sigma_{xa} = K_f \frac{M}{Z_{\text{net}}} = 2.07 \frac{150}{3.31(10^{-6})} = 93.8(10^6)\,\text{Pa} = 93.8\text{ MPa}$$

$$\begin{cases} M_m = 0 \\ T_m = 0 \end{cases}$$

$$\tau_{xya} = K_{fs} \frac{TD}{2J_{\text{net}}} = 1.61 \frac{120(42)(10^{-3})}{2(155)(10^{-9})} = 26.2(10^6)\,\text{Pa} = 26.2\text{ MPa}$$

omponent σ'_m is zero. The alternating component σ'_a is given by

$$\sigma'_a = (\sigma_{xa}^2 + 3\tau_{xya}^2)^{1/2} = [93.8^2 + 3(26.2^2)]^{1/2} = 104\text{ MPa}$$

ersed loading, the fatigue factor of safety n_f is

$$n_f = \frac{S_e}{\sigma'_a} = \frac{148}{104} = 1.42$$

ctor of safety is

$$n_y = \frac{S_y}{\sigma'_a} = \frac{370}{104} = 3.56$$

$T_a = 70$, $T_m = 90$, $M_m = 150$, $M_a = 0$

$$\tau_{xya} = K_{fs}\frac{T_a D}{2J_{\text{net}}} = 1.61\frac{70(42)(10^{-3})}{2(155)(10^{-9})} = 15.3(10^6)\text{Pa} = 15.3\text{ MPa}$$

$$\tau_{xym} = K_{fs}\frac{T_m D}{2J_{\text{net}}} = 1.61\frac{90(42)(10^{-3})}{2(155)(10^{-9})} = 19.7(10^6)\text{Pa} = 19.7\text{ MPa}$$

s component σ_{xm} is

$\sigma_{xa} = 0$

$$\sigma_{xm} = K_f\frac{M_m}{Z_{\text{net}}} = 2.07\frac{150}{3.31(10^{-6})} = 93.8(10^6)\text{Pa} = 93.8\text{ MPa}$$

nponents σ'_a and σ'_m, from Equations (6–66) and (6–67), are

$$\sigma'_a = [3(15.3)^2]^{1/2} = 26.5\text{ MPa}$$

$$\sigma'_m = [93.8^2 + 3(19.7)^2]^{1/2} = 99.8\text{ MPa}$$

$$n_f = \left(\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}}\right)^{-1} = \left(\frac{26.5}{148} + \frac{99.8}{440}\right)^{-1} = 2.46$$

تکلیف فصل ششم

6-10

6-17

6-28

6-35

6-36