



رباتیک پیشرفته سینماتیک، سینماتیک معکوس

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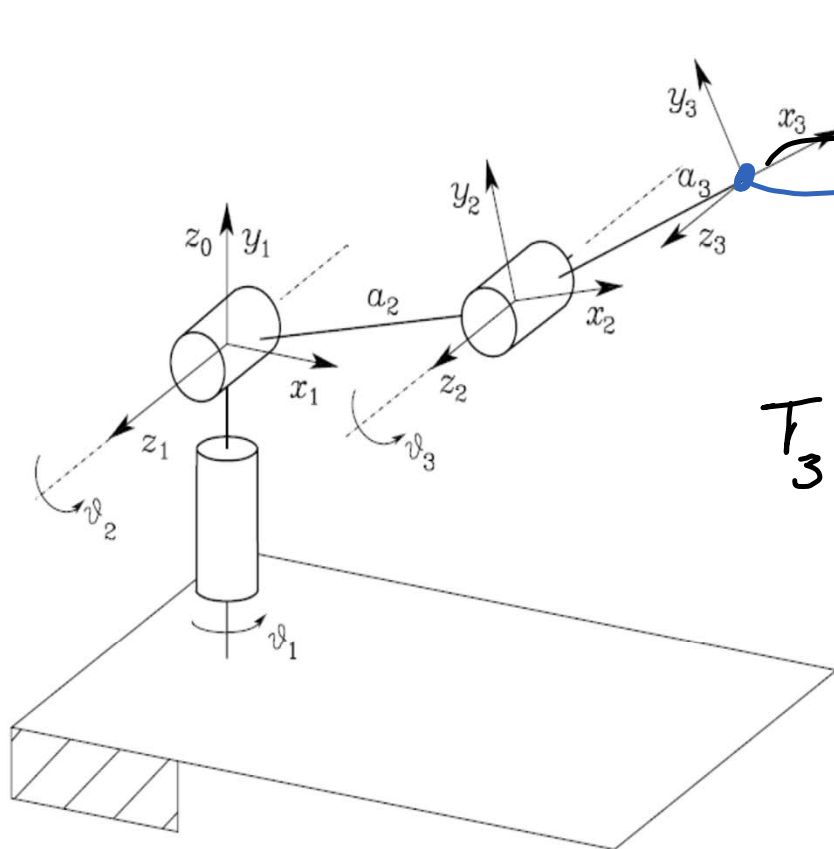


Fig. 2.23. Anthropomorphic arm

$$T_3^0 = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & c_1(a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & s_1(a_2 c_2 + a_3 c_{23}) \\ s_{23} & c_{23} & 0 & a_2 s_2 + a_3 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} c_1(a_2 c_2 + a_3 c_{23}) = x \\ s_1(a_2 c_2 + a_3 c_{23}) = y \\ a_2 s_2 + a_3 s_{23} = z \end{cases}$$



$$\begin{cases} \textcircled{1} \quad c_1 (a_2 c_2 + a_3 c_{23}) = x \\ \textcircled{2} \quad s_1 (a_2 c_2 + a_3 c_{23}) = y \\ \textcircled{3} \quad a_2 s_2 + a_3 s_{23} = z \end{cases}$$

$$c_2 c_{23} + s_2 s_{23} = \cos(\theta_3 + \theta_2 - \theta_1)$$

$$\begin{aligned} \textcircled{1}^2 + \textcircled{2}^2 + \textcircled{3}^2 &\Rightarrow c_1^2 (a_2 c_2 + a_3 c_{23})^2 + s_1^2 (a_2 c_2 + a_3 c_{23})^2 \\ &+ (a_2 s_2 + a_3 s_{23})^2 = (a_2 c_2 + a_3 c_{23})^2 + (a_2 s_2 + a_3 s_{23})^2 \\ &= \underbrace{a_2^2 c_1^2 + a_3^2 c_{23}^2}_{a_2^2 + a_3^2} + \underbrace{a_2^2 s_1^2 + a_3^2 s_{23}^2}_{2a_2 a_3 c_3} + 2a_2 a_3 c_2 c_{23} + 2a_2 a_3 s_2 s_{23} \\ &= a_2^2 + a_3^2 + 2a_2 a_3 c_3 = x^2 + y^2 + z^2 \end{aligned}$$

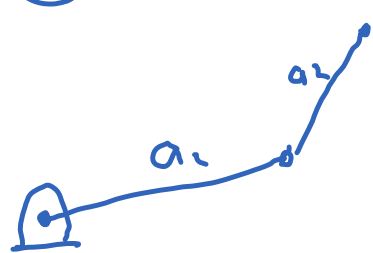


$$a_2^2 + a_3^2 + 2a_2a_3c_3 = x^2 + y^2 + z^2$$

$$\Rightarrow \cos \theta_3 = \frac{x^2 + y^2 + z^2 - a_2^2 - a_3^2}{2a_2a_3} \quad \textcircled{I}, \quad \sin \theta_3 = \pm \sqrt{1 - \cos^2 \theta_3} \quad \textcircled{II}$$

$$\theta_3 = \text{atan2}(\pm \textcircled{II}, \textcircled{I}) \quad , \quad -1 \leq \textcircled{I} \leq 1$$

آر (I) خارج از محدوده قابل تغییر باشد یعنی اینکه بعد از x, y, z خارج از محدوده در است.



$$\begin{bmatrix} x \\ y \end{bmatrix}$$

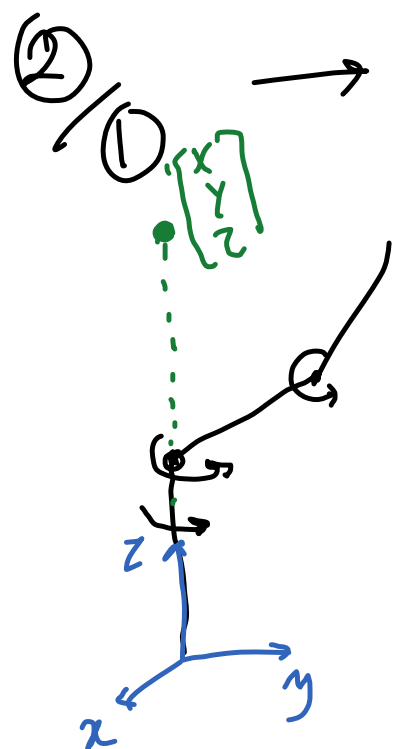


$$\textcircled{1}^2 + \textcircled{2}^2 \Rightarrow (a_2 c_2 + a_3 c_{23})^2 = x^2 + y^2$$

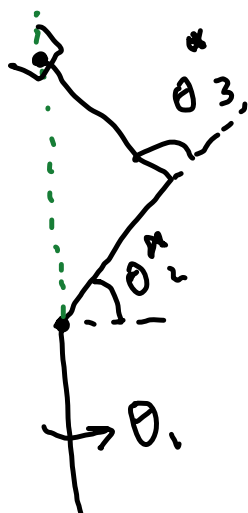
$$\Rightarrow \begin{cases} a_2 c_2 + a_3 c_{23} = \pm \sqrt{x^2 + y^2} = \pm W \\ a_2 s_2 + a_3 s_{23} = Z \end{cases}$$

$$\Rightarrow \begin{cases} a_2 c_2 + a_3 (c_2 c_3 - s_2 s_3) = \pm W \Rightarrow C_2 = \checkmark \\ a_2 s_2 + a_3 (s_2 c_3 + C_2 s_3) = Z \rightarrow S_2 = \checkmark \\ C_2 = \checkmark \end{cases}$$

$$\rightarrow \theta_2 = \text{atan2}(s_2, c_2) \checkmark$$



$$\theta_1 = \arctan^2(y, x)$$

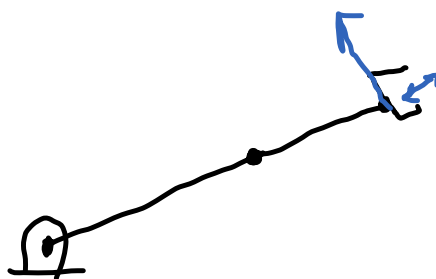


$$\theta_1 = 0, \theta_2^*, \theta_3^*$$

$$\theta_1 = 30, \dots \sim$$

$$\theta_1 = 90, \dots \sim$$

$$\theta_2 = 0$$



Problem, Home work 2

2.1

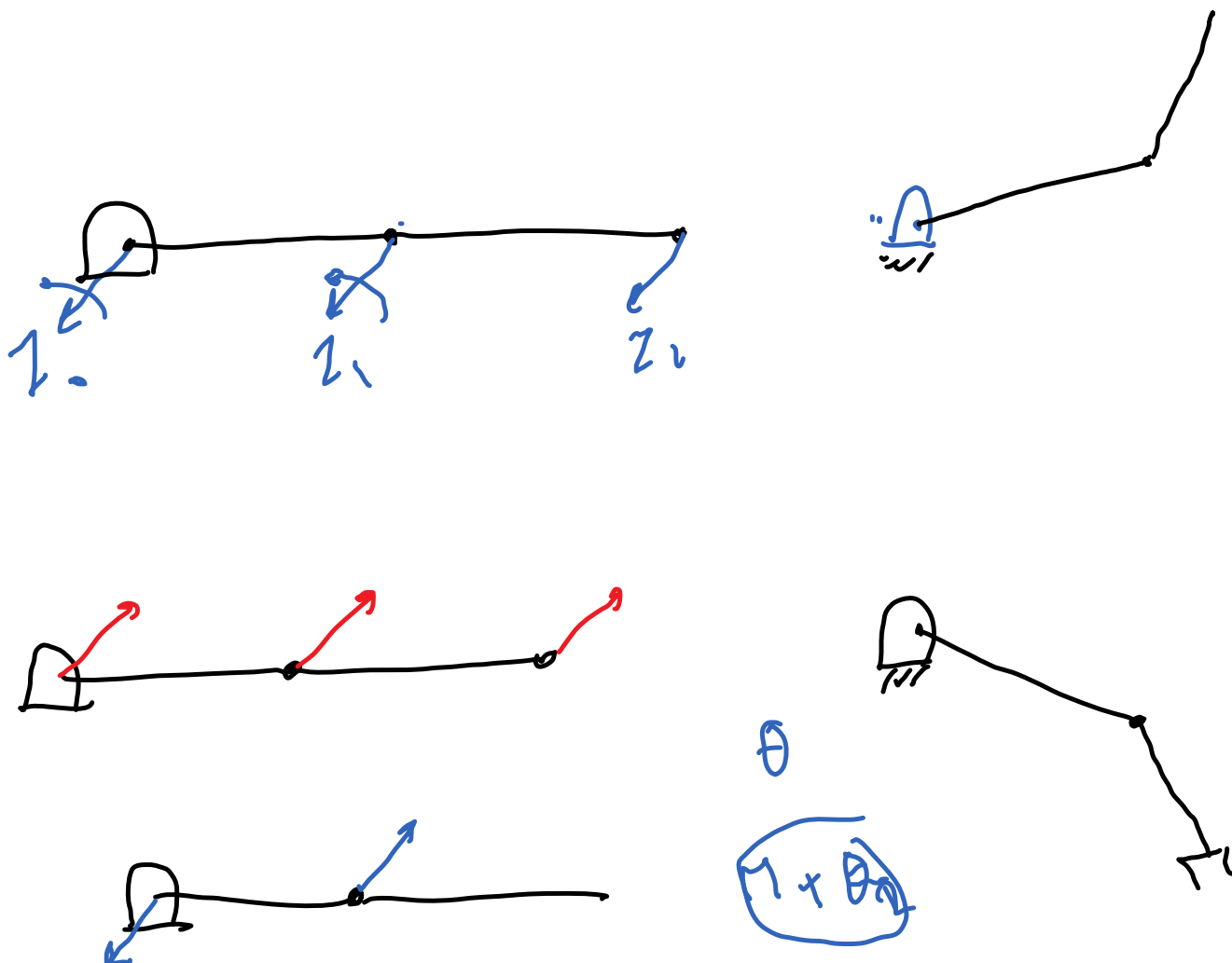
2.12

2.13

2.17

2.19

2.20





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