



رباتیک پیشرفته

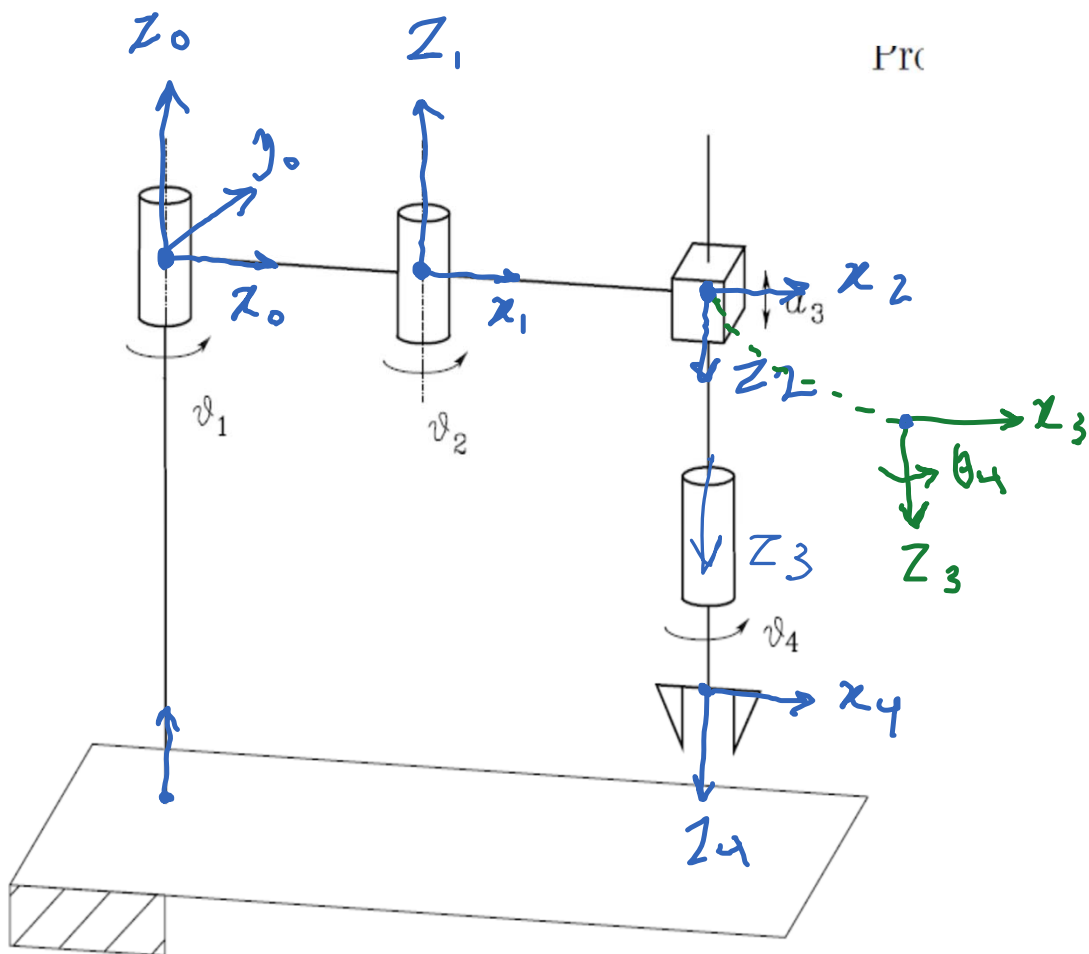
سینماتیک دیفرانسیلی

ژاکوبین هندسی

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PRC

$$\alpha_1 = 0, \alpha_2 = 180$$

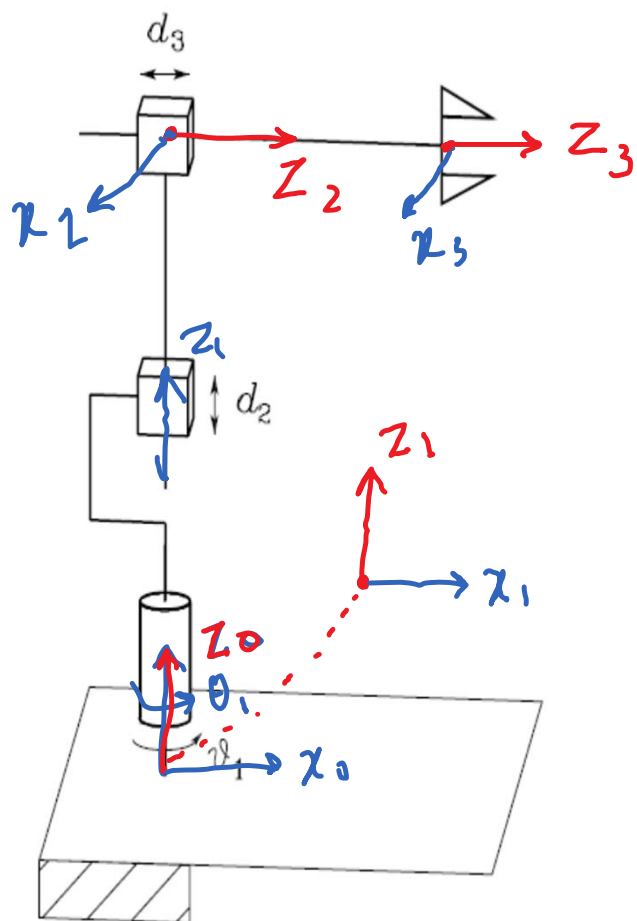
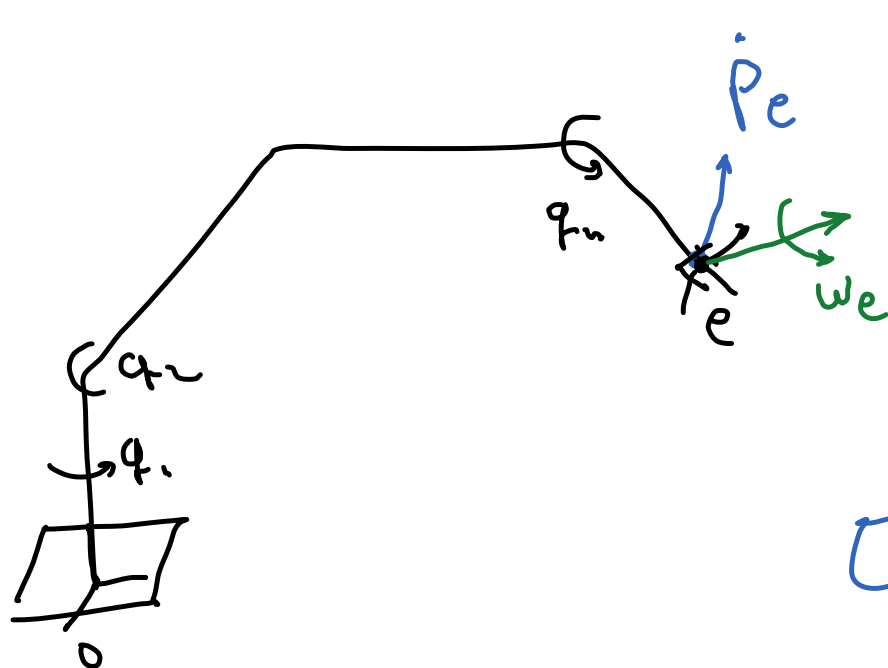


Fig. 2.35. Cylindrical arm



$$T_e^o = \begin{bmatrix} R_e^o & p_e \\ 0 & 1 \end{bmatrix}$$

p_e بردار پوزیشن
 R_e ماتریس چرخش

$$p_e = K(q) \rightarrow$$

در سینماتیک دینامیکی می‌فهمیم رابطه بین سرعت مطلق و زاویه ابزار انتهایی ($\dot{p}_e$)
 رابطه سرعت به متغیرهای مفصل ($\dot{q}$) حساب می‌شود



$$\begin{cases} \dot{p}_e = J_p \dot{q} \\ \omega_e = J_o \dot{q} \end{cases}, \quad v_e = \begin{bmatrix} \dot{p}_e \\ \omega_e \end{bmatrix}, \quad v_e \in \mathbb{R}^{6 \times 1}$$

$$\begin{matrix} x \\ y \\ z \end{matrix} \quad p_e = K(q) \rightarrow \dot{p}_e = \left(\frac{\partial K}{\partial q} \right) \dot{q}$$

$$K \in \mathbb{R}^{3 \times 1}, \quad q \in \mathbb{R}^{n \times 1} \rightarrow J_p \in \mathbb{R}^{3 \times n}$$

$$\omega_e = h(q) \rightarrow J_o \in \mathbb{R}^{3 \times n}$$

$$v_e = J \dot{q}, \quad J \in \mathbb{R}^{6 \times n}$$

6×1 $n \times 1$

J : Geometric Jacobian

$$J = \begin{bmatrix} J_p \\ J_v \end{bmatrix}$$

J ماتریس ژاکوبین سرعت e را، سرعت
مفصل مرتبه می‌گیرد.

In summary, the Jacobian in (3.5) can be partitioned into the (3×1) column vectors J_{Pi} and J_{Oi} as

$$J = \begin{bmatrix} J_{P1} & \dots & J_{Pn} \\ J_{O1} & & J_{On} \end{bmatrix}, \quad (3.29)$$

where

$$\begin{bmatrix} J_{Pi} \\ J_{Oi} \end{bmatrix} = \begin{cases} \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & \text{for a } \textit{prismatic} \text{ joint} \\ \begin{bmatrix} z_{i-1} \times (p_e - p_{i-1}) \\ z_{i-1} \end{bmatrix} & \text{for a } \textit{revolute} \text{ joint.} \end{cases} \quad (3.30)$$

z_{i-1} is given by the third column of the rotation matrix R_{i-1}^0 , i.e.,

$$z_{i-1} = \underbrace{R_1^0(q_1) \dots R_{i-1}^{i-2}(q_{i-1})}_{R_{i-1}^0} z_0 \quad (3) \quad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

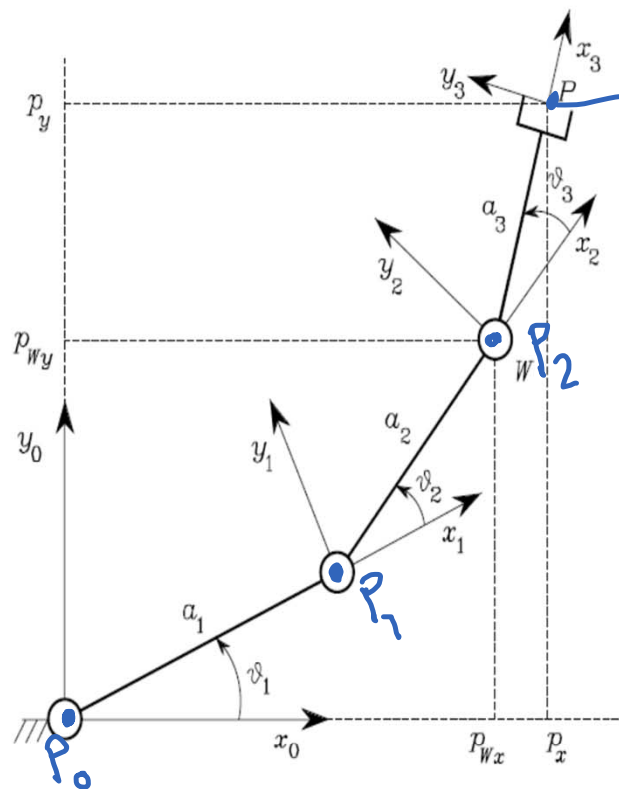
$$\tilde{p}_e = \underbrace{A_1^0(q_1) \dots A_n^{n-1}(q_n)}_{T_n^0} \tilde{p}_0 \quad (3.32)$$

where $\tilde{p}_0 = [0 \ 0 \ 0 \ 1]^T$ allows the selection of the fourth column.

$$\tilde{p}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

به سندن هم میزنس A_n^0

$$\tilde{p}_{i-1} = \underbrace{A_1^0(q_1) \dots A_{i-1}^{i-2}(q_{i-1})}_{A_{i-1}^0} \tilde{p}_0.$$



$$A_i^{i-1}(\vartheta_i) = \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad i = 1, 2, 3.$$

Link	a_i	α_i	d_i	ϑ_i
1	a_1	0	0	ϑ_1
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

$\rightarrow A_1^0$
 $\rightarrow A_2^1$
 $\rightarrow A_3^2$

①

② ، A_1^0 ، T_2^0 ، T_3^0

$T_2^0 = A_1^0 A_2^1$ ، $T_3^0 = T_2^0 A_3^2$ و

$T_2^0 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1 c_1 + a_2 c_{12} \\ s_{12} & c_{12} & 0 & a_1 s_1 + a_2 s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Annotations: z_2 points to the third column, p_2 points to the fourth column.

$T_3^0(q) = A_1^0 A_2^1 A_3^2 = \begin{bmatrix} c_{123} & -s_{123} & 0 & a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ s_{123} & c_{123} & 0 & a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Annotations: $p_3 = p_e$ points to the fourth column, z_1 points to the third column, p_1 points to the fourth column.

$T_1^0 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Annotations: z_1 points to the third column, p_1 points to the fourth column.

$$J = \begin{bmatrix} J_p \\ J_o \end{bmatrix} = \begin{bmatrix} J_{p1} & J_{p2} & J_{p3} \\ J_{o1} & J_{o2} & J_{o3} \end{bmatrix}$$

$\uparrow \quad \quad \uparrow \quad \quad \uparrow$
 $R \quad \quad R \quad \quad R$

$$J_{p_i} = \left[z_{i-1} \times (p_e - p_{i-1}) \right]$$

بدای مسافت (معدنی)

$$J_{o_i} = \left[z_{i-1} \right]$$

$$J = \begin{bmatrix} z_0 \times (p_e - p_0) & z_1 \times (p_e - p_1) & z_2 \times (p_e - p_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$



$$P_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad P_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix}$$

$$Z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



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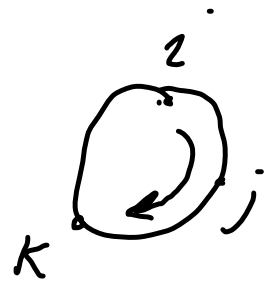
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$$Z_0 \times (P_e - P_o) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix}$$

$$= (JK) \times \left[(a_1 c_1 + a_2 c_{12} + a_3 c_{123}) i' + (a_1 s_1 + a_2 s_{12} + a_3 s_{123}) j' \right]$$

$$= (a_1 c_1 + a_2 c_{12} + a_3 c_{123}) j' - (a_1 s_1 + a_2 s_{12} + a_3 s_{123}) i'$$

$$= \begin{bmatrix} -(a_1 s_1 + a_2 s_{12} + a_3 s_{123}) \\ (a_1 c_1 + a_2 c_{12} + a_3 c_{123}) \\ 0 \end{bmatrix}$$



$$J = \begin{bmatrix} -(a_1 s_1 + a_2 s_{12} + a_3 s_{123}) & -a_1 s_{12} - a_3 s_{123} & -a_3 s_{123} \\ (a_1 c_1 + a_2 c_{12} + a_3 c_{123}) & a_2 c_{12} + a_3 c_{123} & a_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$



$$P_e = \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix}, \quad P_e = K(q)$$

$$P_e = K(\theta_1, \theta_2, \theta_3) \Rightarrow \dot{P}_e = \frac{\partial K}{\partial \theta_1} \dot{\theta}_1 + \frac{\partial K}{\partial \theta_2} \dot{\theta}_2 + \frac{\partial K}{\partial \theta_3} \dot{\theta}_3$$

$$= \begin{bmatrix} \frac{\partial K}{\partial \theta_1} & \frac{\partial K}{\partial \theta_2} & \frac{\partial K}{\partial \theta_3} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$= J_p \dot{q}$$

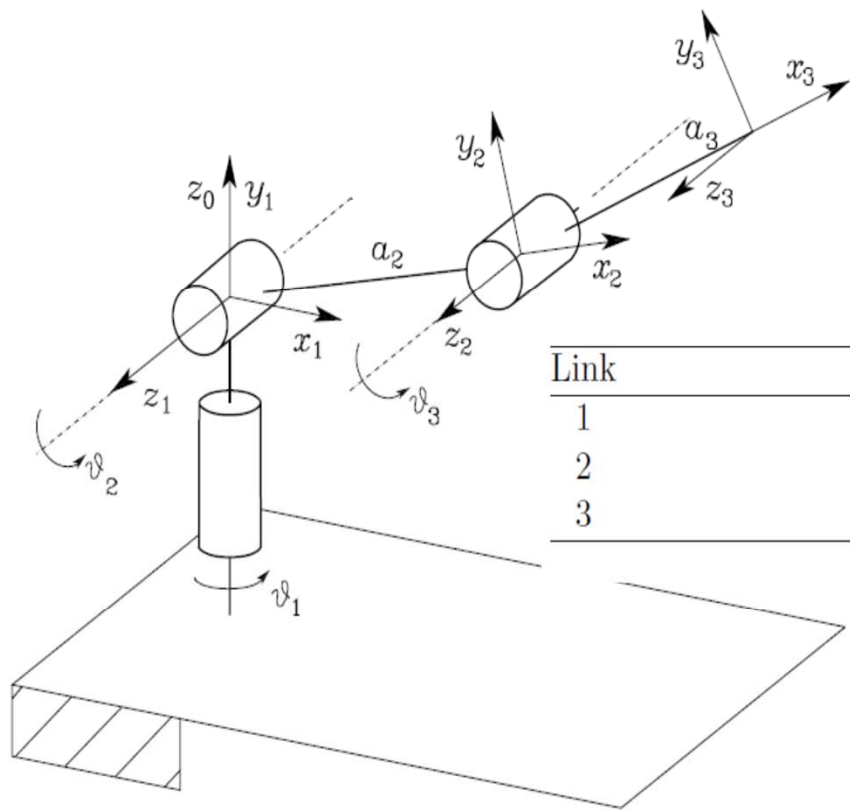
J_p



$$P_e = \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix}$$

$$J_p = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} - a_3 s_{123} & -a_2 s_{12} - a_3 s_{123} & -a_3 s_{123} \\ a_1 c_1 + a_2 c_{12} + a_3 c_{123} & a_2 c_{12} + a_3 c_{123} & a_3 c_{123} \\ 0 & 0 & 0 \end{bmatrix}$$

$\frac{\partial K}{\partial \theta_1}$ $\frac{\partial K}{\partial \theta_2}$ $\frac{\partial K}{\partial \theta_3}$



Link	a_i	α_i	d_i	ϑ_i
1	0	$\pi/2$	0	ϑ_1
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

$\rightarrow A_1^0$
 $\rightarrow A_2^1$
 $\rightarrow A_3^2$

$$T_1^0 = A_1^0$$

$$T_2^0 = A_1^0 A_2^1$$

$$T_3^0 = T_2^0 A_3^2$$

Fig. 2.23. Anthropomorphic arm

$$A_1^0(\vartheta_1) = \begin{bmatrix} c_1 & 0 & \boxed{s_1} & 0 \\ s_1 & 0 & \boxed{-c_1} & 0 \\ 0 & 1 & \boxed{0} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1^0 A_2^1$$

$$A_i^{i-1}(\vartheta_i) = \begin{bmatrix} c_i & -s_i & 0 & a_i c_i \\ s_i & c_i & 0 & a_i s_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad i = 2, 3.$$

$$T_3^0(q) = A_1^0 A_2^1 A_3^2 = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & c_1 (a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & s_1 (a_2 c_2 + a_3 c_{23}) \\ s_{23} & c_{23} & 0 & a_2 s_2 + a_3 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^0 = A_1^0 A_2^1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_1 c_2 & -c_1 s_2 & \overset{z_2}{\boxed{s_1}} & a_2 c_1 c_2 \\ s_1 c_2 & -s_1 s_2 & \boxed{-c_1} & a_2 s_1 c_2 \\ s_2 & c_2 & \boxed{0} & a_2 s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$Z_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 1 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

$$T_3^0(q) = A_1^0 A_2^1 A_3^2 = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & c_1(a_2 c_2 + a_3 c_{23}) \\ s_1 c_{23} & -s_1 s_{23} & -c_1 & s_1(a_2 c_2 + a_3 c_{23}) \\ s_{23} & c_{23} & 0 & a_2 s_2 + a_3 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_e = \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} \\ a_2 s_1 c_2 + a_3 s_1 c_{23} \\ a_2 s_2 + a_3 s_{23} \end{bmatrix}$$



$$P_e = \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} \\ a_2 s_1 c_2 + a_3 s_1 c_{23} \\ a_2 s_2 + a_3 s_{23} \end{bmatrix} \quad J_p = \begin{bmatrix} \frac{\partial P_e}{\partial \theta_1} & \frac{\partial P_e}{\partial \theta_2} & \frac{\partial P_e}{\partial \theta_3} \end{bmatrix}$$

$$J_p = \begin{bmatrix} -a_2 c_2 s_1 - a_2 s_1 c_{23} & -a_2 c_1 s_2 - a_3 c_1 s_{23} & -a_3 c_1 s_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & -a_2 s_1 s_2 - a_3 s_1 s_{23} & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \end{bmatrix}$$



$$J = \begin{bmatrix} -a_2 c_2 s_1 - a_2 s_1 c_{23} & -a_1 c_1 s_2 - a_3 c_1 s_{23} & -a_3 c_1 s_{23} \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & -a_1 s_1 s_2 - a_3 s_1 s_{23} & -a_3 s_1 s_{23} \\ 0 & a_2 c_2 + a_3 c_{23} & a_3 c_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} J_p \\ J_v \end{bmatrix} = \begin{bmatrix} \sim & \sim & \sim \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} -s_1(a_2c_2 + a_3c_{23}) & -c_1(a_2s_2 + a_3s_{23}) & -a_3c_1s_{23} \\ c_1(a_2c_2 + a_3c_{23}) & -s_1(a_2s_2 + a_3s_{23}) & -a_3s_1s_{23} \\ 0 & a_2c_2 + a_3c_{23} & a_3c_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} J_{Pi} \\ J_{Oi} \end{bmatrix} = \begin{cases} \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} & \text{for a prismatic joint} \\ \begin{bmatrix} z_{i-1} \times (p_e - p_{i-1}) \\ z_{i-1} \end{bmatrix} & \text{for a revolute joint.} \end{cases} \quad (3.30)$$

مثال RPR

مختصات اول دورانی
~ درم تغیری
~ ~ دورانی

$$J = \begin{bmatrix} z_0 \times (p_e - p_0) & z_1 & z_2 \times (p_e - p_1) \\ z_0 & 0 & z_2 \end{bmatrix}$$

$$p_e(\theta_1, d_2, \theta_3) \rightarrow J_p = \begin{bmatrix} \frac{\partial p_e}{\partial \theta_1} & \frac{\partial p_e}{\partial d_2} & \frac{\partial p_e}{\partial \theta_3} \end{bmatrix}$$



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