



مکاترونیک

مروری بر دینامیک و مدلسازی دینامیکی

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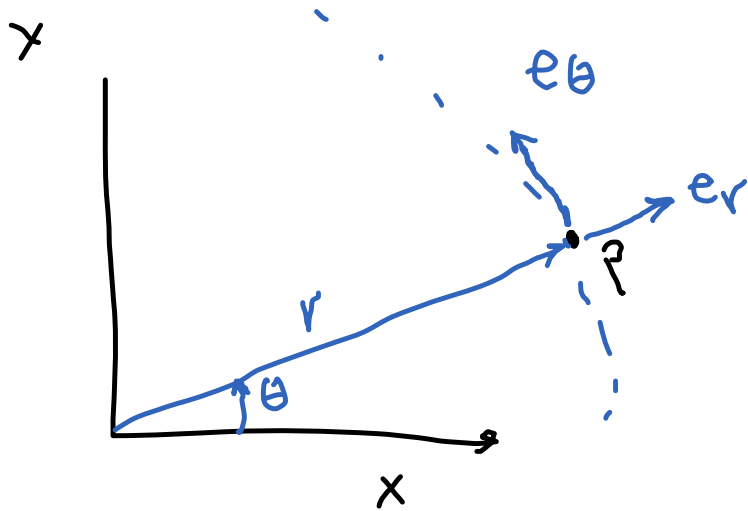
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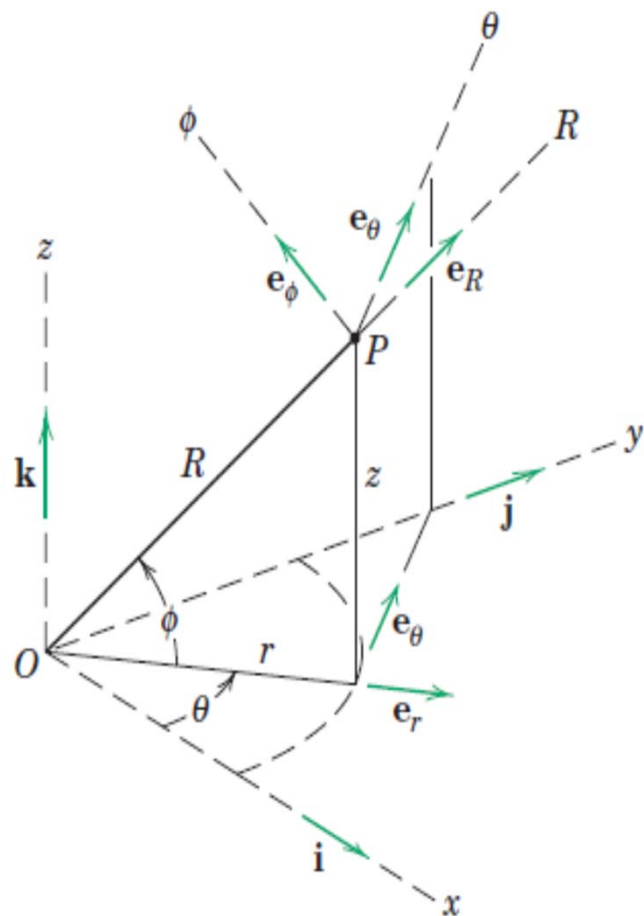


سریع دست - از مضامین قبلی

$$\mathbf{v} = \dot{r} \mathbf{e}_r + r \dot{\theta} \mathbf{e}_\theta$$

$$\mathbf{a} = (\ddot{r} - r \dot{\theta}^2) \mathbf{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \mathbf{e}_\theta$$





$$\dot{v}_r = \dot{R} e_r$$

$$\dot{v}_\phi = R \dot{\phi} e_\phi$$

$$\dot{v}_\theta = r \dot{\theta} + R \dot{\phi} \sin \phi e_\theta$$



تحلیل حرکت اجسام با روش رفرنس عامل حرکت Kinetics

قانون دوم نیوتن

$$\sum \vec{F} = m\vec{a}$$

نیروی آزاد جنبشی

استخراج معادلات دینامیکی سیستم

- انتخاب یک است. حرکت

- جدا کردن هر جسم از اجزای دیگر و رسم دیاگرام آزاد جنبشی

- نوشتن معادلات $\sum F = ma$ در جهات مختلف

- حذف نیدرهای داخلی و استخراج معادلات دینامیکی

نیروی آزاد جنبشی

$$\sum \vec{F} = m\vec{a} \quad \xrightarrow{\text{کارتزین}} \quad \begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \\ \sum F_z = ma_z \end{cases}$$

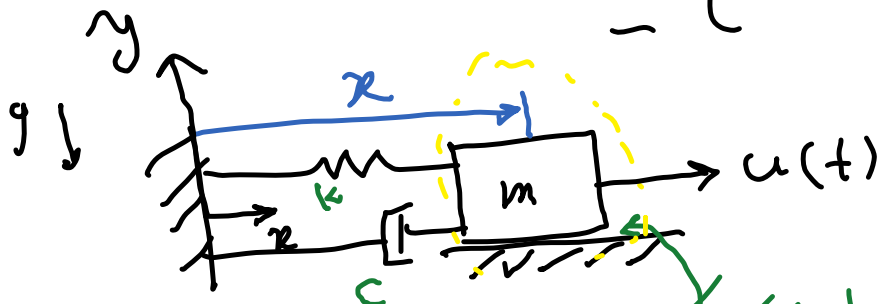
$$\left. \begin{array}{l} \text{تangent} \\ \text{normal} \end{array} \right\} \begin{cases} \sum F_t = ma_t \\ \sum F_n = ma_n \end{cases}$$

$$\left. \begin{array}{l} \text{radial} \\ \text{tangential} \end{array} \right\} \begin{cases} \sum F_r = ma_r \\ \sum F_\theta = ma_\theta \\ \sum F_z = ma_z \end{cases}$$

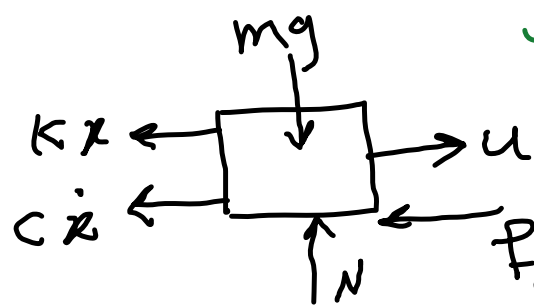


مدل سازی
معادلات این سیستم یک جسم در محدوده استوار است.

۲. مقدار جابجایی نسبت به طول آزارنده



لغزش در سطح



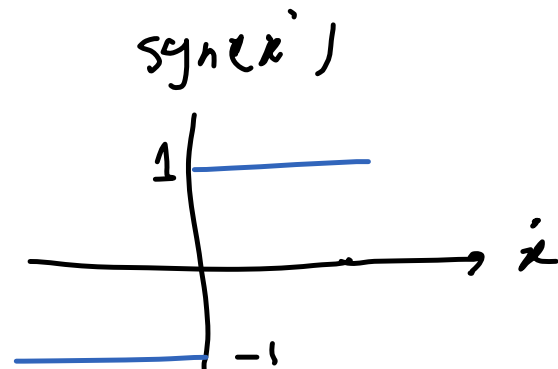
$$F_F = \mu N \operatorname{sgn}(\dot{x})$$

دینامیک صلبی

mech

دینامیک تار،
 $\sum F$

$$\boxed{} \rightarrow m\ddot{x}$$



$$\begin{cases} \sum F_x = ma_x \rightarrow u - kx - c\dot{x} - \mu N \operatorname{sgn}(\dot{x}) = m\ddot{x} \\ \sum F_y = ma_y \rightarrow N - mg = 0 \Rightarrow N = mg \end{cases}$$

$$\Rightarrow u - kx - c\dot{x} - \mu mg \operatorname{sgn}(\dot{x}) = m\ddot{x}$$

استخراج معادله دینامیکی سیستم کامل شد.



$$u - k - c\dot{x} - \mu mg \operatorname{sgn}(\dot{x}) = m\ddot{x}$$

حل و تحلیل، معادله دیفرانسیل ODE ←

N-ODE ← حل عددی، numerical sol. ←

راشونک 45

دست. معادله دیفرانسیل غیر خطی مرتبه اول را حل می کند. → ode 45 →

$$\dot{x} = f(x)$$

$$u - kx - c\dot{x} - \mu mg \operatorname{sgn}(\ddot{x}) = m\ddot{x}$$

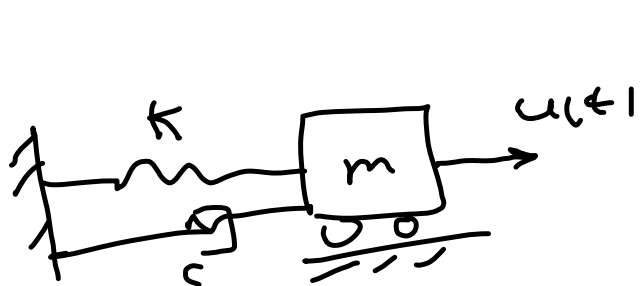
نشان می‌دهد که سیستم در حالت دائمی می‌رسد. state space

$$\begin{cases} x = x_1 \\ \dot{x} = x_2 \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m} [u - kx_1 - cx_2 - \mu mg \operatorname{sgn}(x_2)] \end{cases}$$

$$x(0) = x_1(0) \quad \rightarrow \quad [x_1(0) \quad x_2(0)]$$

$$\dot{x}(0) = x_2(0)$$

$$\omega_n = \sqrt{\frac{k}{m}}, \quad c, \mu = 0$$



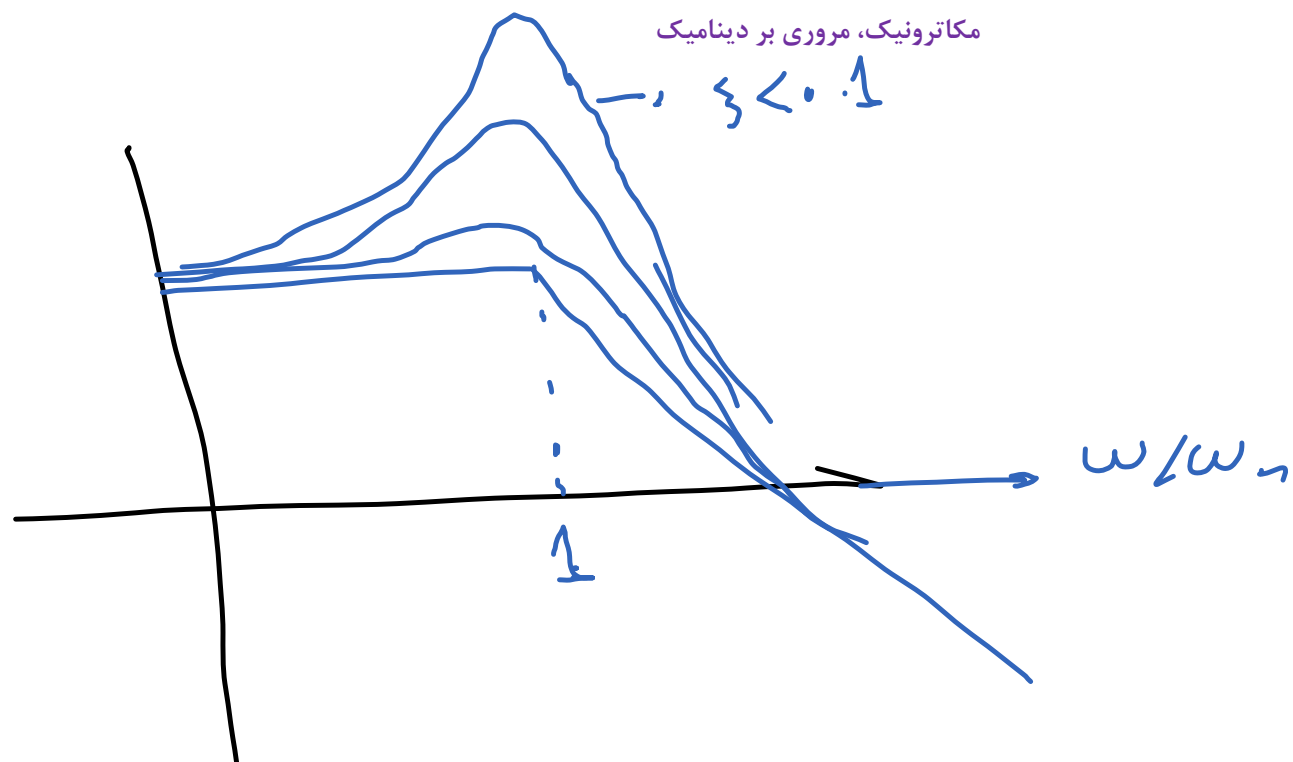
$$\omega_n = \sqrt{\frac{k}{m}}$$

$$u(t) = \sin \omega t, \quad \omega = \omega_n$$

اگر فرکانس نزدیک به فرکانس طبیعی سیستم نزدیک شود چه اتفاقی می افتد؟

$$m\ddot{x} + c\dot{x} + kx = u \rightarrow U(s)$$

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + cs + k}$$



SAMPLE PROBLEM 3/1

A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in the hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and the upward velocity v of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.

Solution. The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

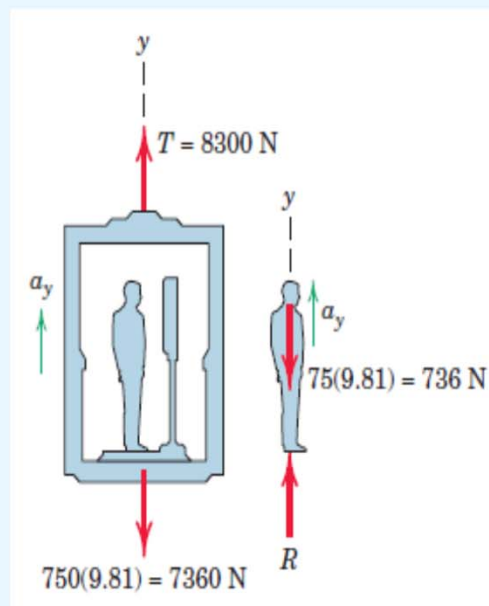
$$[\Sigma F_y = ma_y] \quad 8300 - 7360 = 750a_y \quad a_y = 1.257 \text{ m/s}^2$$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R to this action is shown on the free-body diagram of the man alone together with his weight, and the equation of motion for him gives

$$1 \quad [\Sigma F_y = ma_y] \quad R - 736 = 75(1.257) \quad R = 830 \text{ N} \quad \text{Ans.}$$

The velocity reached at the end of the 3 seconds is

$$[\Delta v = \int a \, dt] \quad v - 0 = \int_0^3 1.257 \, dt \quad v = 3.77 \text{ m/s} \quad \text{Ans.}$$

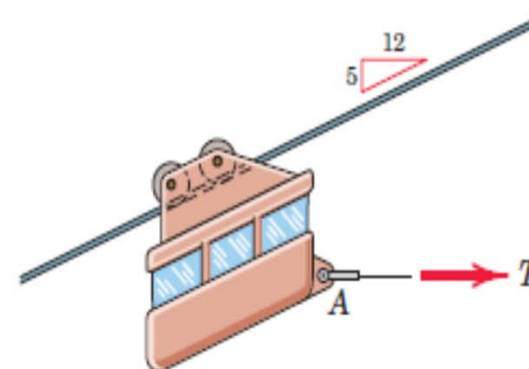


Helpful Hint

- 1 If the scale were calibrated in kilograms, it would read $830/9.81 = 84.6$ kg which, of course, is not his true mass since the measurement was made in a noninertial (accelerating) system. *Suggestion:* Rework this problem in U.S. customary units.

SAMPLE PROBLEM 3/2

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at A. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force P exerted by the supporting cable on the wheels.



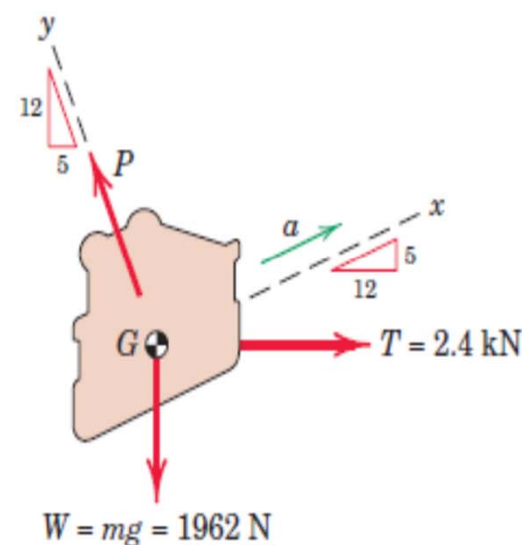
Solution. The free-body diagram of the car and wheels taken together and treated as a particle discloses the 2.4-kN tension T , the weight $W = mg = 200(9.81) = 1962$ N, and the force P exerted on the wheel assembly by the cable.

The car is in equilibrium in the y -direction since there is no acceleration in this direction. Thus,

$$[\Sigma F_y = 0] \quad P - 2.4\left(\frac{5}{13}\right) - 1962\left(\frac{12}{13}\right) = 0 \quad P = 2.73 \text{ kN} \quad \text{Ans.}$$

1 In the x -direction the equation of motion gives

$$[\Sigma F_x = ma_x] \quad 2400\left(\frac{12}{13}\right) - 1962\left(\frac{5}{13}\right) = 200a \quad a = 7.30 \text{ m/s}^2 \quad \text{Ans.}$$



SAMPLE PROBLEM 3/3

The 250-lb concrete block *A* is released from rest in the position shown and pulls the 400-lb log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at *B*.

Solution. The motions of the log and the block *A* are clearly dependent. Although by now it should be evident that the acceleration of the log up the incline is half the downward acceleration of *A*, we may prove it formally. The constant total length of the cable is $L = 2s_C + s_A + \text{constant}$, where the constant accounts for the cable portions wrapped around the pulleys. Differentiating twice with respect to time gives $0 = 2\ddot{s}_C + \ddot{s}_A$, or

$$0 = 2a_C + a_A$$

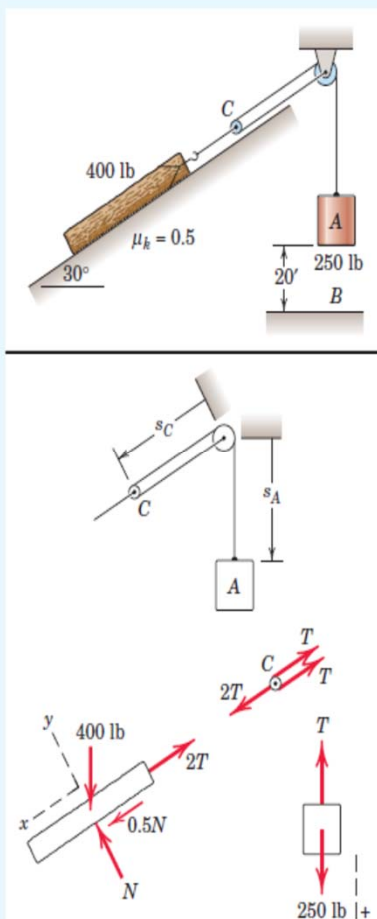
We assume here that the masses of the pulleys are negligible and that they turn with negligible friction. With these assumptions the free-body diagram of the pulley *C* discloses force and moment equilibrium. Thus, the tension in the cable attached to the log is twice that applied to the block. Note that the accelerations of the log and the center of pulley *C* are identical.

The free-body diagram of the log shows the friction force $\mu_k N$ for motion up the plane. Equilibrium of the log in the *y*-direction gives

$$[\Sigma F_y = 0] \quad N - 400 \cos 30^\circ = 0 \quad N = 346 \text{ lb}$$

and its equation of motion in the *x*-direction gives

$$[\Sigma F_x = ma_x] \quad 0.5(346) - 2T + 400 \sin 30^\circ = \frac{400}{32.2} a_C$$



For the block in the positive downward direction, we have

$$[+ \downarrow \Sigma F = ma] \quad 250 - T = \frac{250}{32.2} a_A$$

Solving the three equations in a_C , a_A , and T gives us

$$a_A = 5.83 \text{ ft/sec}^2 \quad a_C = -2.92 \text{ ft/sec}^2 \quad T = 205 \text{ lb}$$

For the 20-ft drop with constant acceleration, the block acquires a velocity

$$[v^2 = 2ax] \quad v_A = \sqrt{2(5.83)(20)} = 15.27 \text{ ft/sec} \quad \text{Ans.}$$

Helpful Hints

- 1 The coordinates used in expressing the final kinematic constraint relationship must be consistent with those used for the kinetic equations of motion.
- 2 We can verify that the log will indeed move up the ramp by calculating the force in the cable necessary to initiate motion from the equilibrium condition. This force is $2T = 0.5N + 400 \sin 30^\circ = 373 \text{ lb}$ or $T = 186.5 \text{ lb}$, which is less than the 250-lb weight of block *A*. Hence, the log will move up.

- 3/9** A man pulls himself up the 15° incline by the method shown. If the combined mass of the man and cart is 100 kg, determine the acceleration of the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.

