



دینامیک پیشرفته سینماتیک

دکتر امین نیکوبین

دانشگاه سمنان، دانشکده مهندسی مکانیک

anikoobin@semnan.ac.ir



دسته‌های مختلف از قائم و مماس

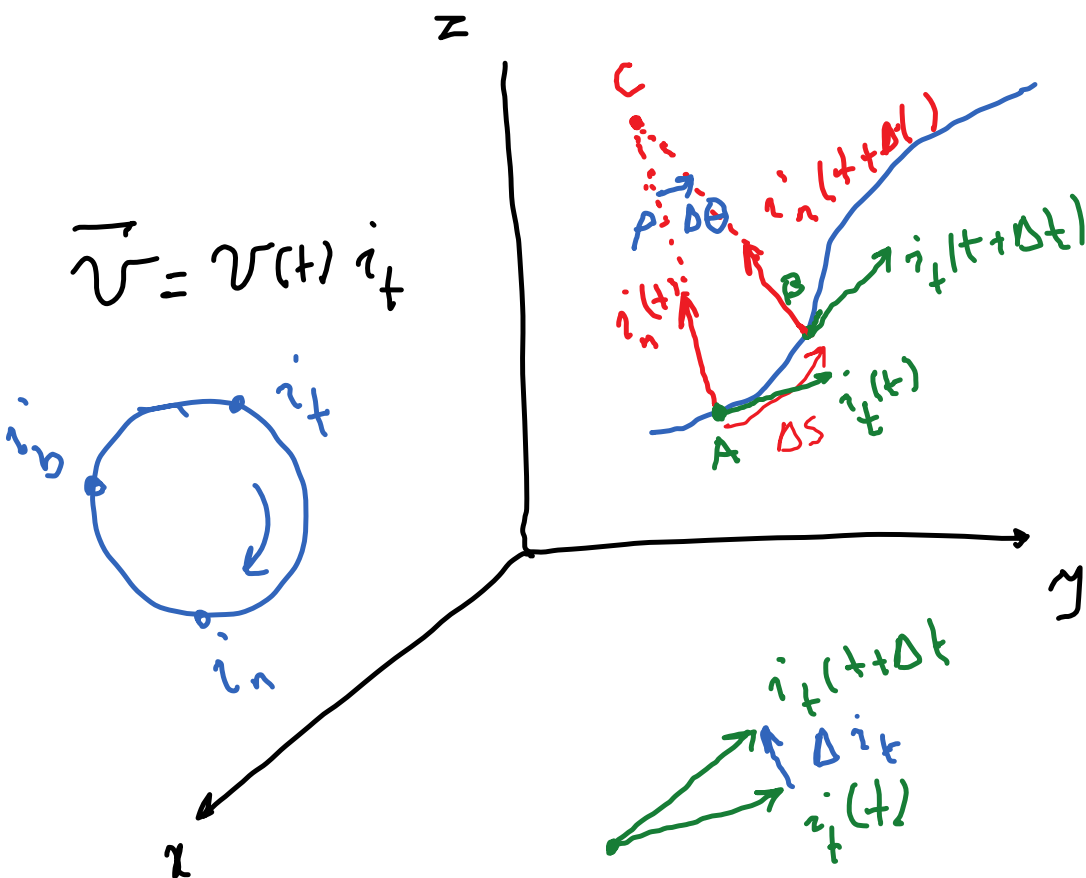
Tangential and Normal

جهت و کدین یک بردار مماس به i_t : سرعت
کدین

صفحه بوسان

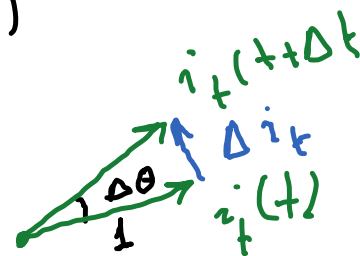
osculatory plane

صفحه‌ای است که از A و i_t می‌گذرد و
به موازات $i_t(t+\Delta t)$ می‌باشد
 $i_n(t)$ در صفحه بوسان قرار دارد.

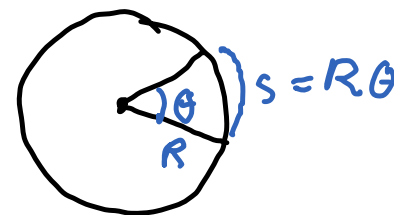




$$\lim_{\Delta t \rightarrow 0} \frac{\Delta i_t}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{i(t + \Delta t) - i(t)}{\Delta t} = \frac{1(\dot{\theta}) i_n(t)}{\Delta t}$$



$$\Rightarrow \frac{di_t}{dt} = \frac{d\theta}{dt} i_n(t)$$



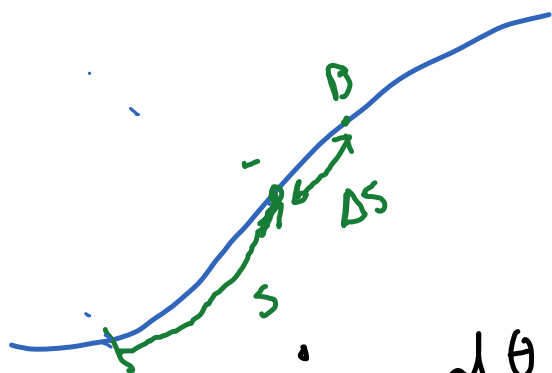


$$\vec{v} = v \dot{\vec{r}}_t, \quad v = \frac{ds}{dt} = \dot{s}, \quad \dot{v} = \frac{d^2s}{dt^2} = \ddot{s}$$

$$v_{av} = \frac{\Delta s}{\Delta t}$$

برای محاسبه

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v} \dot{\vec{r}}_t + v \ddot{\vec{r}}_t$$



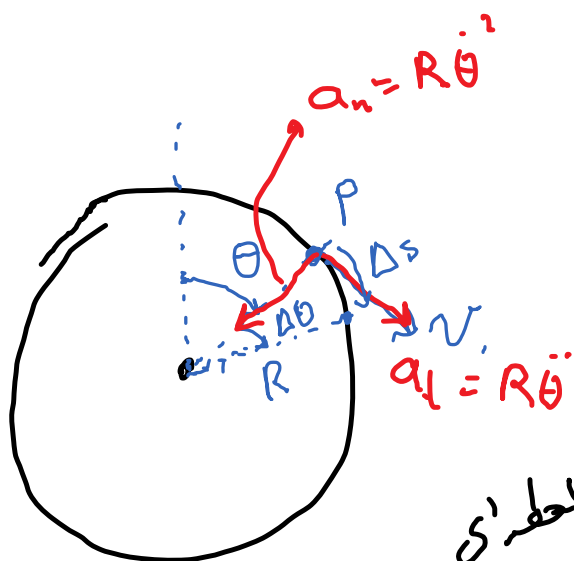
$$\ddot{\vec{r}}_t = \frac{d\theta}{dt} \dot{\vec{r}}_n = \frac{d\theta}{ds} \frac{ds}{dt} \dot{\vec{r}}_n = \frac{1}{\rho} v \dot{\vec{r}}_n$$

$$\Delta s = \rho \Delta \theta \Rightarrow \Delta s = \rho \Delta \theta \Rightarrow ds = \rho d\theta$$

$$\Rightarrow \frac{d\theta}{ds} = \frac{1}{\rho}$$

$$a = \frac{dv}{dt} i_t + \frac{v^2}{\rho} i_n = a_t i_t + a_n i_n$$

حالت خاص ثابت $\rho =$

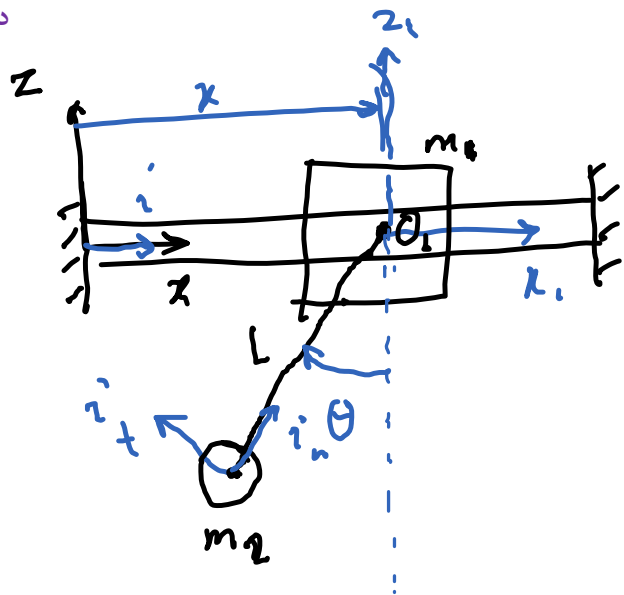


$$v = \frac{\Delta s}{\Delta t} = \frac{R \Delta \theta}{\Delta t}$$

$$v = R \frac{d\theta}{dt} = R \dot{\theta} = R \omega$$

$$a = R \ddot{\theta} i_t + \frac{R \dot{\theta}^2}{R} i_n = R \ddot{\theta} i_t + R \dot{\theta}^2 i_n$$

$$= R \alpha i_t + R \omega^2 i_n$$



مثال. سرعت و شتاب جیم m_2

$$a_2 = a_1 + a_{2/1}$$

$$a_1 = \ddot{x} i_1$$

$$a_{2/1} = L \ddot{\theta} i_{t_2} + L \dot{\theta}^2 i_{n_2}$$

$$a_2 = \ddot{x} i_1 + L \ddot{\theta} i_{t_2} + L \dot{\theta}^2 i_{n_2}$$

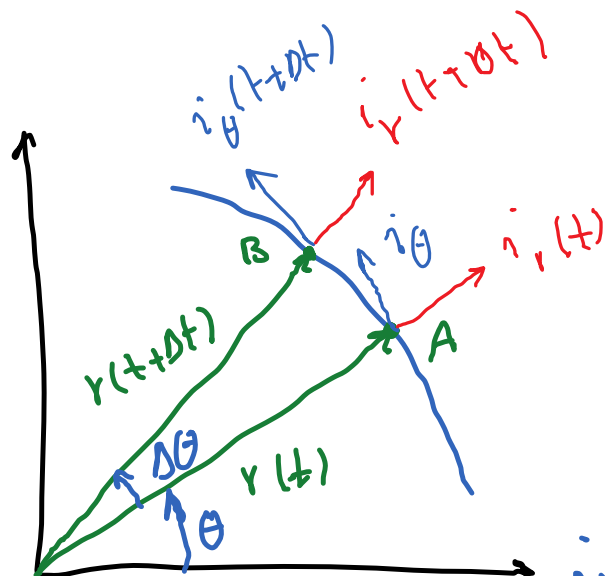
$$v_2 = \dot{x} i_1 + L \dot{\theta} i_{t_2}$$



مختصات قطبی
صفای

Polar

(r, θ)



$$\vec{r} = r \dot{i}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r} \dot{i}_r + r \dot{\dot{i}}_r$$

$$\Delta \dot{i}_r = \dot{i}_r(t+dt) - \dot{i}_r(t) = 1(\Delta\theta) \dot{i}_\theta$$

$$\Rightarrow \dot{\dot{i}}_r = \dot{\theta} \dot{i}_\theta$$

$$\Delta \dot{i}_\theta = 1(\Delta\theta) (-\dot{i}_r)$$

$$\Rightarrow \dot{\dot{i}}_\theta = -\dot{\theta} \dot{i}_r$$



$\dot{i}_r(t+dt)$

$\dot{i}_r(t)$

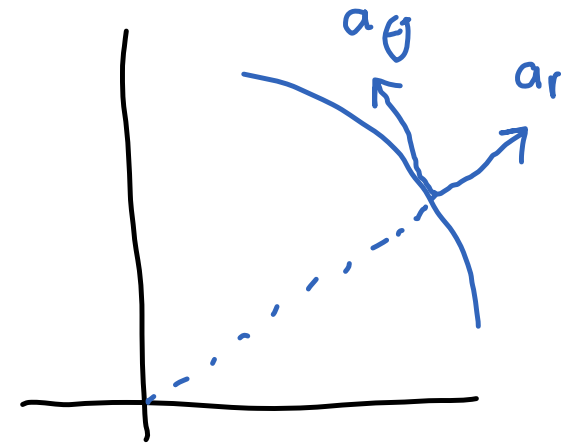
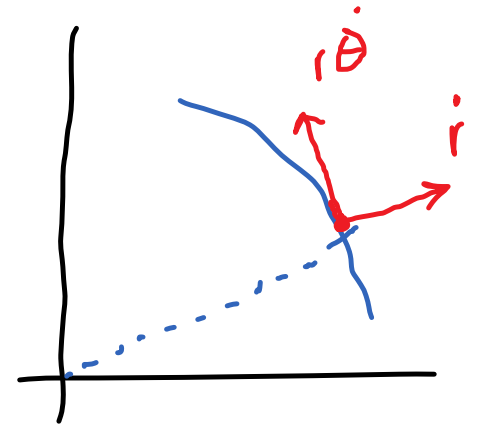
$$\vec{v} = \dot{r} \hat{i}_r + r \dot{\theta} \hat{i}_\theta = \dot{r} \hat{i}_r + r \dot{\theta} \hat{i}_\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r} \hat{i}_r + \dot{r} \dot{\theta} \hat{i}_\theta + \dot{r} \dot{\theta} \hat{i}_\theta + r \ddot{\theta} \hat{i}_\theta + r \dot{\theta} \dot{\theta} \hat{i}_\theta$$

$$= \ddot{r} \hat{i}_r + \underbrace{\dot{r} \dot{\theta} \hat{i}_\theta}_{\text{cross term}} + \underbrace{\dot{r} \dot{\theta} \hat{i}_\theta}_{\text{cross term}} + r \ddot{\theta} \hat{i}_\theta - r \dot{\theta}^2 \hat{i}_r$$

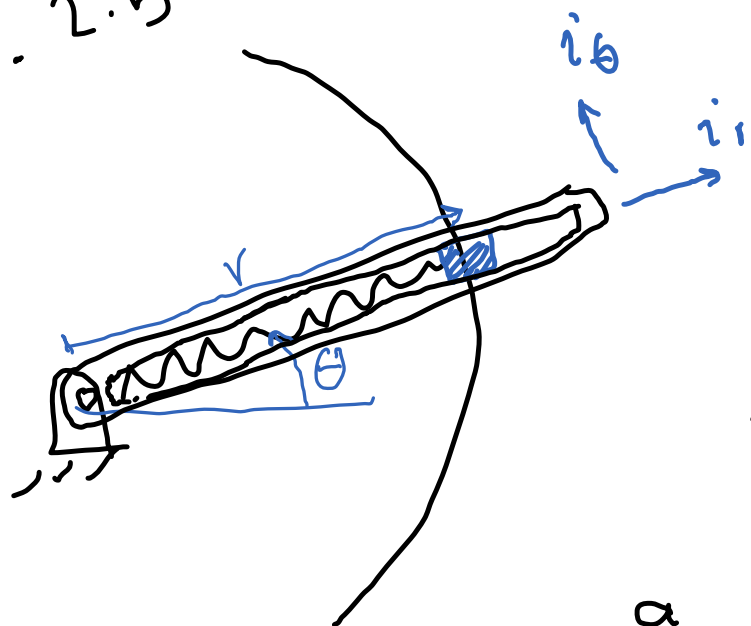
$$= (\ddot{r} - r \dot{\theta}^2) \hat{i}_r + (2 \dot{r} \dot{\theta} + r \ddot{\theta}) \hat{i}_\theta$$

$$= a_r \hat{i}_r + a_\theta \hat{i}_\theta$$





Ex. 2.5



$$\theta = c \sin \omega t$$

$$r = \frac{a}{1 + \theta}$$

$$v = \dot{r} i_r + r \dot{\theta} i_{\theta}$$

$$r = \frac{a}{1 + c \sin \omega t}, \quad \dot{r} = \frac{-ac\omega \cos \omega t}{(1 + c \sin \omega t)^2}$$

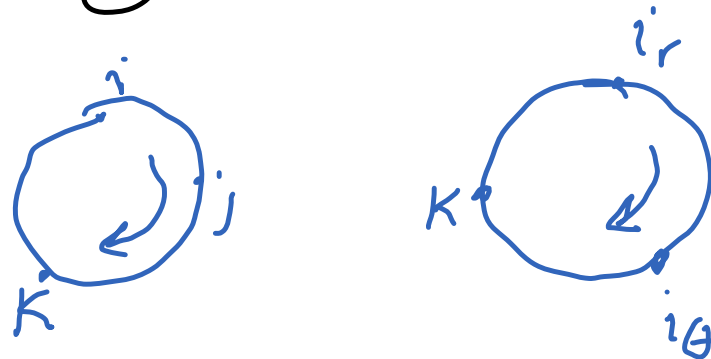
$$\dot{\theta} = c\omega \cos \omega t$$

$$\ddot{\theta} = -c\omega^2 \sin \omega t$$

منبع:
ω



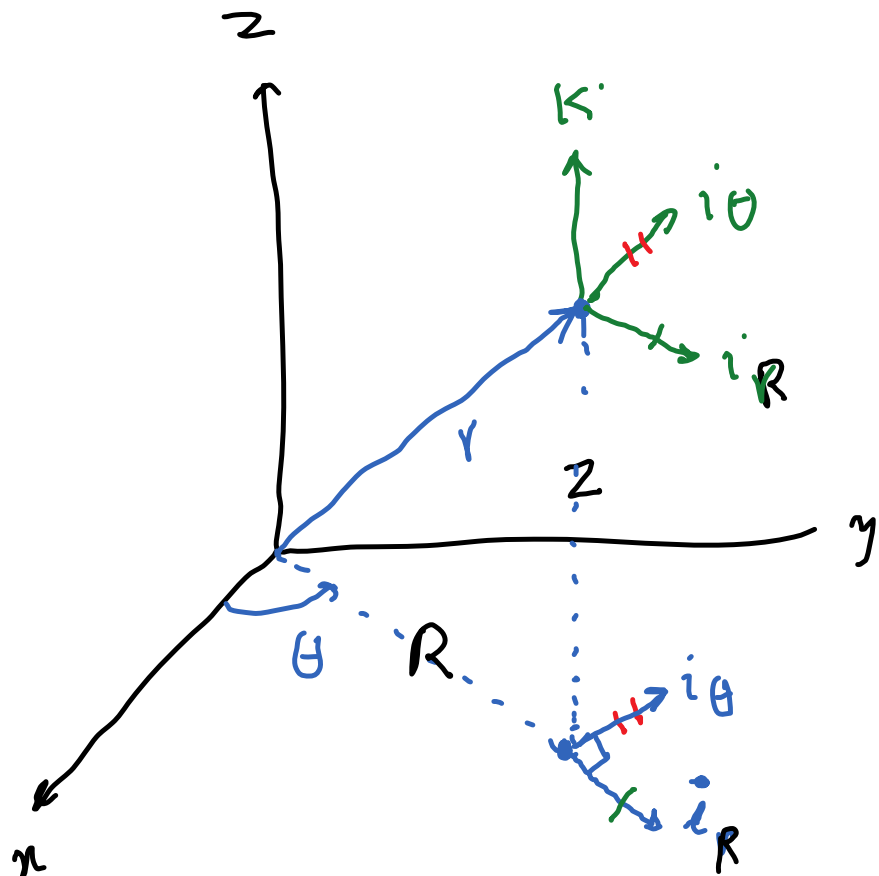
مقدمات استاتیسی



$$\vec{r} = R \vec{i}_R + z \vec{k}$$

$$\vec{v} = \dot{\vec{r}} = \dot{R} \vec{i}_R + R \dot{\theta} \vec{i}_\theta + \dot{z} \vec{k}$$

$$\vec{a} = (\ddot{R} - R \dot{\theta}^2) \vec{i}_R + (R \ddot{\theta} + 2 \dot{R} \dot{\theta}) \vec{i}_\theta + \ddot{z} \vec{k}$$





دینامیک پیشرفته، سینماتیک

دکتر امین نیکوبین