



دینامیک پیشرفته

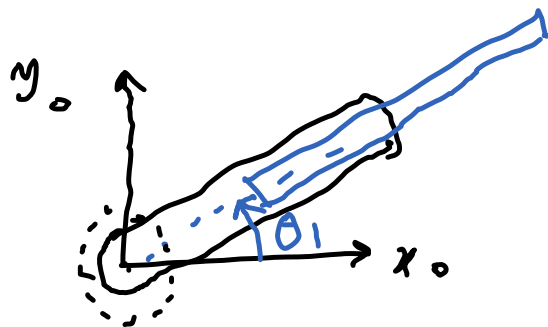
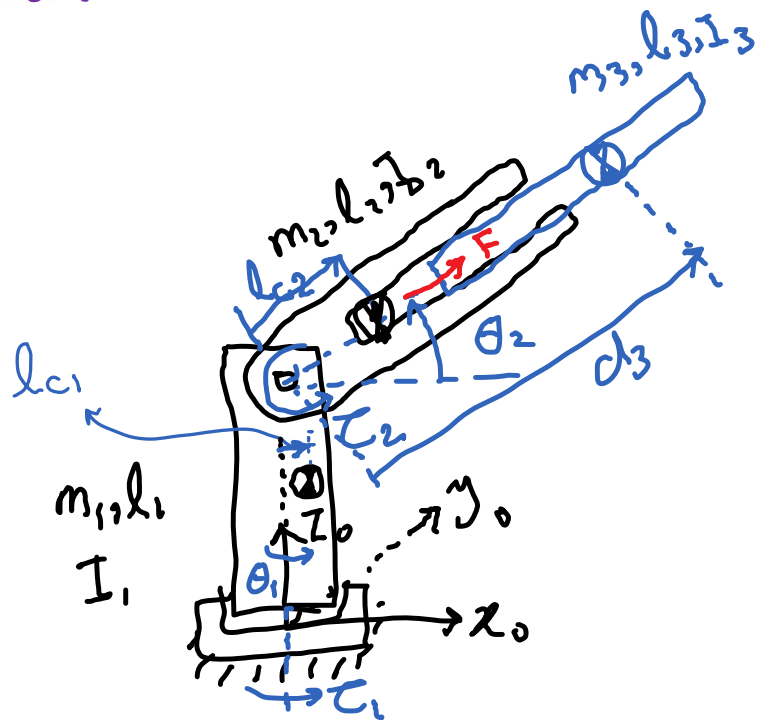
دینامیک لاگرانژین

استخراج معادلات دینامیکی به روش لاگرانژ
ربات سه درجه آزادی فضایی
به همراه شبیه سازی در MATLAB

دکتر امین نیکوبین

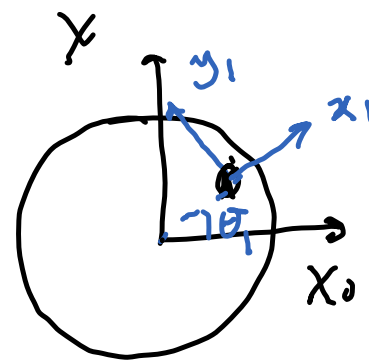
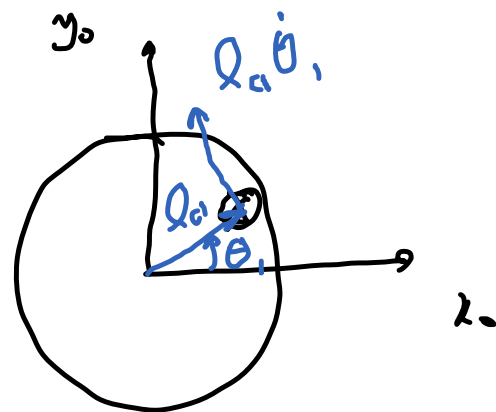
دانشگاه سمنان، دانشکده مهندسی مکانیک

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مسئله انرژی جنبشی

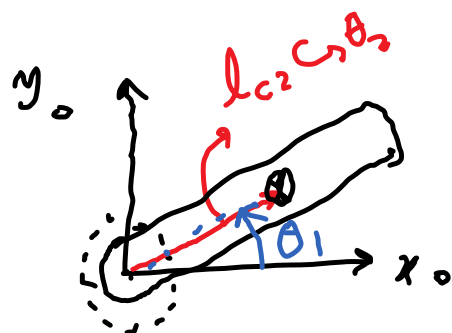
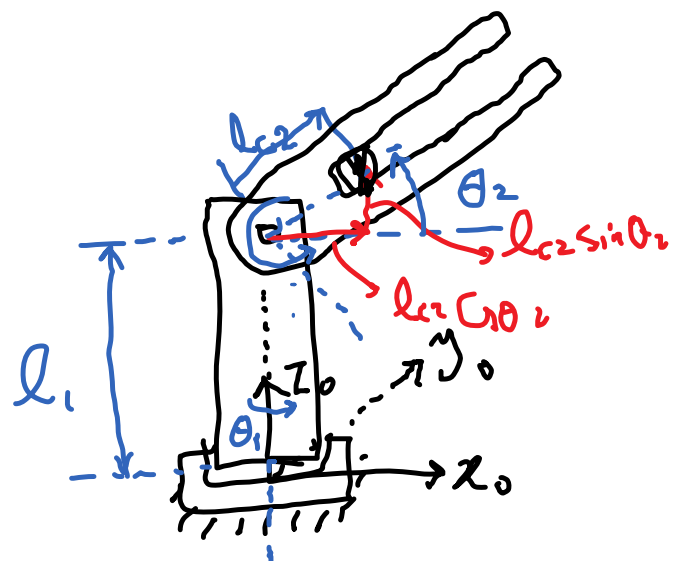
$$v_{C1} = -l_{C1} \dot{\theta}_1 \sin \theta_1 + l_{C1} \dot{\theta}_1 \cos \theta_1$$



$$R_1^0 = R_2(\theta_1)$$

$$v_{C1}^2 = v_{C1x}^2 + v_{C1y}^2 = (l_{C1} \dot{\theta}_1)^2$$

$$\omega_1 = \dot{\theta}_1, K = \begin{bmatrix} 0 \\ \dot{\theta}_1 \end{bmatrix} \textcircled{1}$$

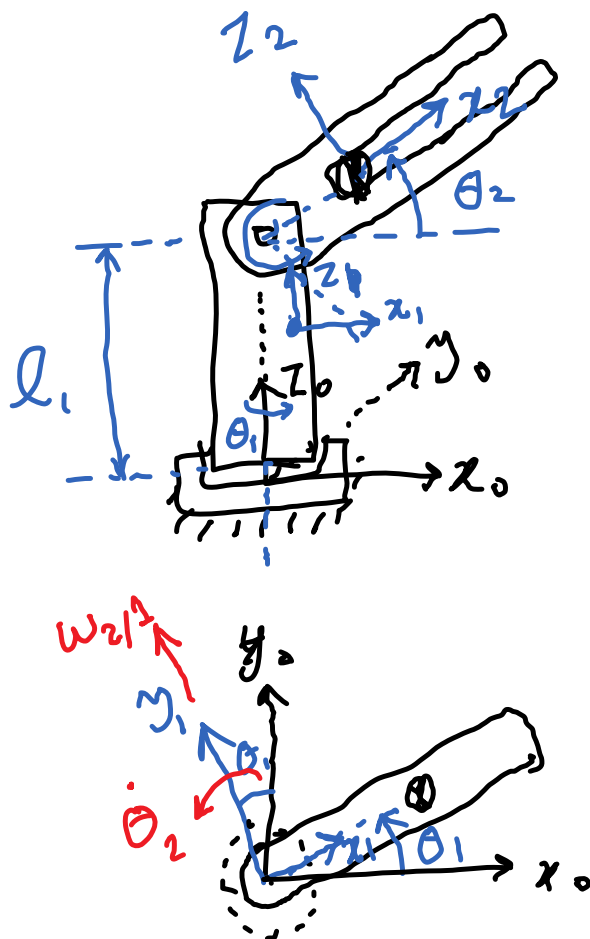


سرعت تغییر موقع

$${}^0r_{C2} = \begin{bmatrix} l_{c2} \cos \theta_2 \cos \theta_1 \\ l_{c2} \cos \theta_2 \sin \theta_1 \\ l_1 + l_{c2} \sin \theta_2 \end{bmatrix}$$

$$v_{C2} = \frac{d}{dt} {}^0r_{C2} = \begin{bmatrix} -l_{c2} \dot{\theta}_2 \sin \theta_2 \cos \theta_1 - l_{c2} \dot{\theta}_1 \cos \theta_2 \sin \theta_1 \\ \dots \\ \dots \end{bmatrix}$$

$$v_{C2}^2 = v_{C2x}^2 + v_{C2y}^2 + v_{C2z}^2$$



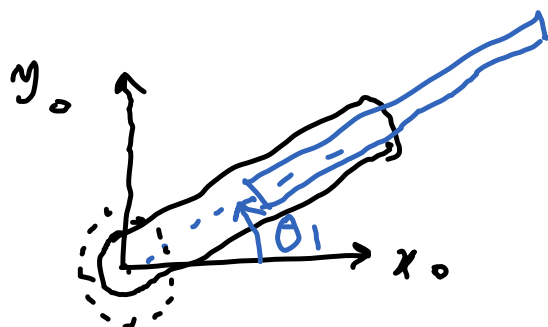
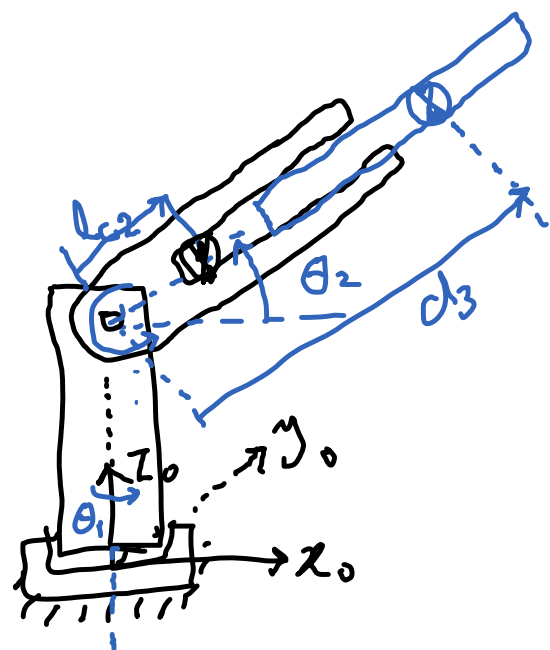
تسک درم حول مصدر y_1 دوران دارد

$$\omega_2 = \omega_1 + \omega_{2/1} = \dot{\theta}_1 k_0 + \dot{\theta}_2 j_1$$

$$\Rightarrow \omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + R_1^0 \begin{bmatrix} 0 \\ \dot{\theta}_2 \\ 0 \end{bmatrix}$$

$$\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta}_2 \\ 0 \end{bmatrix}$$

$$\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} -\dot{\theta}_2 \sin \theta_1 \\ \dot{\theta}_2 \cos \theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\dot{\theta}_2 \sin \theta_1 \\ \dot{\theta}_2 \cos \theta_1 \\ \dot{\theta}_1 \end{bmatrix} \quad (2)$$



نسبت مس

$${}^0r_{C3} = \begin{bmatrix} d_3 \cos \theta_2 \cos \theta_1 \\ d_3 \cos \theta_2 \sin \theta_1 \\ l_1 + l_2 \sin \theta_2 \end{bmatrix}$$

$$v_{C3} = \frac{d}{dt} r_{C3} = \begin{bmatrix} \dot{d}_3 \cos \theta_2 - d_3 \dot{\theta}_1 \sin \theta_2 - d_3 \dot{\theta}_2 \sin \theta_1 \\ \dot{d}_3 \sin \theta_2 + d_3 \dot{\theta}_1 \cos \theta_2 - d_3 \dot{\theta}_2 \cos \theta_1 \\ \dot{l}_2 \cos \theta_2 \end{bmatrix}$$

$$v_{C3}^2 = v_{C3x}^2 + v_{C3y}^2 + v_{C3z}^2 \quad \left| \quad \omega_3 = \omega_2 \right.$$



$$T = T_1 + T_2 + T_3$$

$$T_i = \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} \omega_i^T I_i \omega_i = T_{it} + T_{ir}$$

اثری ناشی از یکیت استرلست
و مسجل از c انتساب نداشت

$$\frac{1}{2} {}^o \omega_i^T {}^o I_i {}^o \omega_i = \frac{1}{2} {}^i \omega_i^T {}^i I_i {}^i \omega_i$$

$${}^o \omega_i = R_i {}^i \omega_i$$

$${}^i \omega_i = R_i^T {}^o \omega_i$$

$$\frac{1}{2} (R_i^T {}^o \omega_i)^T {}^i I_i (R_i {}^i \omega_i)$$

$$= \frac{1}{2} {}^o \omega_i^T \underbrace{R_i^T {}^i I_i R_i}_{{}^o I_i} {}^o \omega_i \Rightarrow \boxed{{}^o I_i = R_i^T {}^i I_i R_i}$$



$$T_{i,r} = \frac{1}{2} \omega_i^T (R_i^0 I_i R_i^{0T}) \omega_i$$

ω_i ها از روابط 1, 2 جابجایی می‌شوند

$$R_1^0 = R_z(\theta_1) = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^1 = R_y(\theta_2) = \begin{bmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{bmatrix} \rightarrow R_2^0 = R_1^0 R_2^1$$

$$R_3^2 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow R_3^0 = R_2^0$$

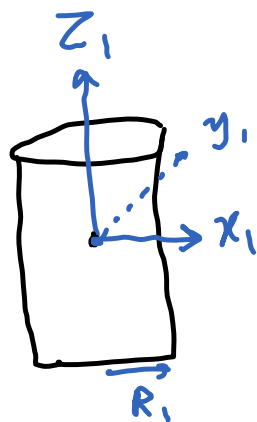
مسئله همان است که در اینجا

در صورتی که I_{z1} ظاهر می شود.

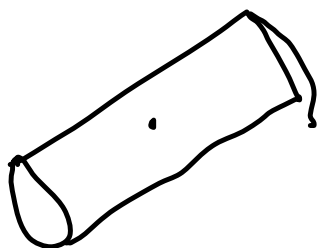
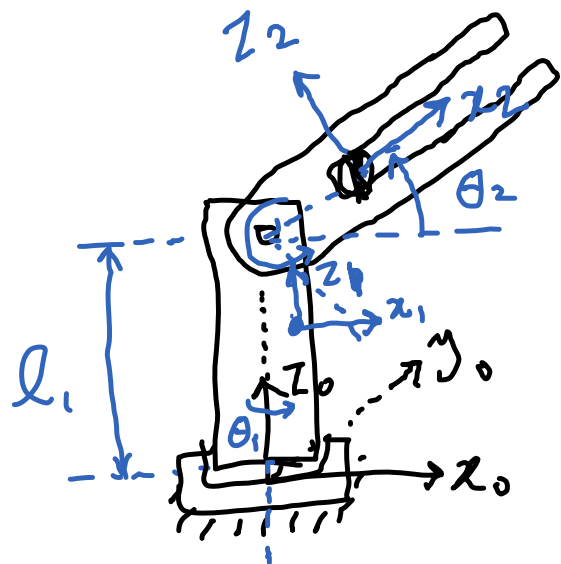
$$I_1 = \begin{bmatrix} I_{x1} & 0 & 0 \\ 0 & I_{y1} & 0 \\ 0 & 0 & I_{z1} \end{bmatrix}$$

فرض شود مرکز جرم روی محور دوران وارد است
($h_{c1} = 0$)

$$I_{z1} = \frac{m_1 R_1^2}{4}$$

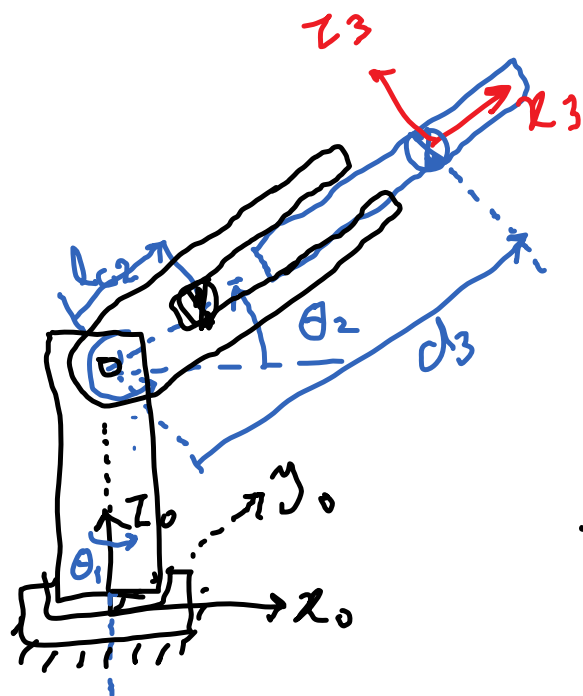


$${}^2I_2 = \begin{bmatrix} I_{x_2} & 0 & 0 \\ 0 & I_{y_2} & 0 \\ 0 & 0 & I_{z_2} \end{bmatrix}$$



اگر تک 2 به صورت یک سطر یک ستاره باشد برابر R_2 و در صورتی که R_2

$${}^2I_2 = \begin{bmatrix} \frac{m_2 R_2^2}{2} & \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} & \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} \\ \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} & \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} & \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} \\ \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} & \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} & \frac{m_2 l_2^2}{12} + \frac{m_2 R_2^2}{4} \end{bmatrix}$$



$${}^2I_3 = {}^3I_3 = \begin{bmatrix} I_{x3} & 0 & 0 \\ 0 & I_{y3} & 0 \\ 0 & 0 & I_{z3} \end{bmatrix}$$

$${}^3I_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m_3 l_3^2}{12} & 0 \\ 0 & 0 & \frac{m_3 l_3^2}{12} \end{bmatrix}$$

انرژی پتانسیل

$$U = U_1 + U_2 + U_3$$

$$U_1 = 0$$

$$U_2 = m_2 g l_{c2} \sin \theta_2$$

$$U_3 = m_3 g d_3 \sin \theta_2$$

$$L = T - U$$

$$\begin{cases} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = \tau_1 \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = \tau_2 \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_3} \right) - \frac{\partial L}{\partial x_3} = F_3 \end{cases}$$



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