دانشگاه سمنان

کنترل مدرن

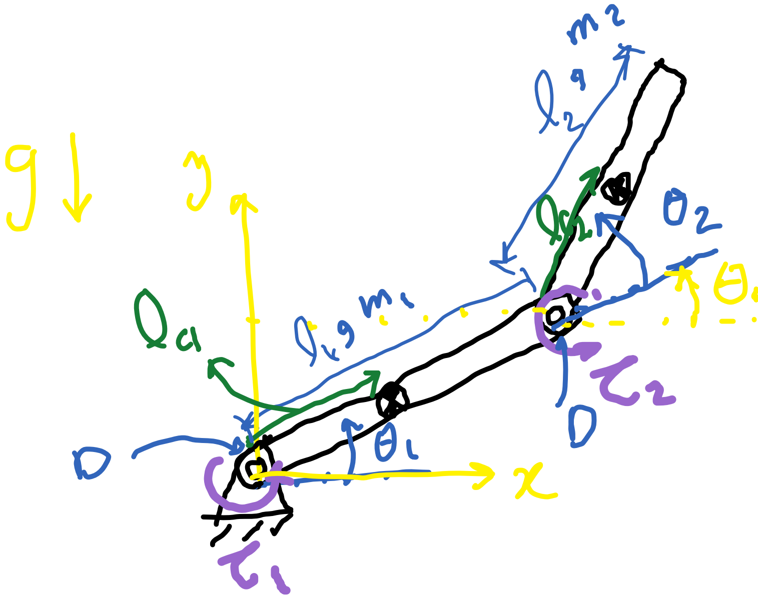
مدل سازی، شبیه سازی و کنترل ربات دو لینکی به همراه کد نویسی در MATLAB

دکتر امین نیکوبین

دانشگاه سمنان، دانشکده مهندسی مکانیک

anikoobin@semnan.ac.ir

معادلات دینامیکی ربات دولینکی



$$q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad , \quad Q = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau - D\dot{q}$$

$$M = \begin{bmatrix} m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2) + 2m_2 l_1 l_{c2}\cos\theta_2 + I_1 + I_2 & m_2 l_{c2}^2 + I_2 + m_2 l_1 l_{c2}\cos\theta_2 \\ m_2 l_{c2}^2 + m_2 l_1 l_{c2}\cos\theta_2 + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$$

$$V = \begin{bmatrix} -2m_2 l_1 l_{c2}\dot{\theta}_1\dot{\theta}_2\sin\theta_2 - m_2 l_1 l_{c2}\dot{\theta}_2^2\sin\theta_2 \\ -m_2 l_1 l_{c2}\dot{\theta}_1\dot{\theta}_2\sin\theta_2 \end{bmatrix}$$

$+$

$\dot{\theta}_1^2$

$$G = \begin{bmatrix} (m_1 l_{c1} + m_2 l_1) g \cos\theta_1 + m_2 g l_{c2} \cos(\theta_1 + \theta_2) \\ m_2 g l_{c2} \cos(\theta_1 + \theta_2) \end{bmatrix}$$

$$\ddot{q} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} , \quad \dot{q} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} , \quad q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

State space معادلات حالت

$$\underset{2\times 2}{M} \; \underset{2\times 1}{\ddot{q}} = \begin{bmatrix} - \; \cdots \\ \\ \cdots \end{bmatrix}_{2\times 1}$$

$$x_1 = \theta_1$$
$$x_2 = \theta_2$$
$$x_3 = \dot{\theta}_1$$
$$x_4 = \dot{\theta}_2$$

$$\begin{cases} x_1 = \theta_1 \\ x_2 = \theta_2 \\ x_3 = \dot{\theta}_1 \\ x_4 = \dot{\theta}_2 \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = M^{-1}(x_1, x_2)\left[\tau - V(x_1, \dots, x_4) - G(x_1, x_2) - D\dot{q}\right] \end{cases}$$

$$M\ddot{q} + V + G = \tau \rightarrow \ddot{q} = M^{-1}(\tau - V - G - D\dot{q})$$

$$D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$$

d_1 و d_2 ضریب اصطکاک ویسکوز در مفصل 1 و 2

$$D\dot{q} = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

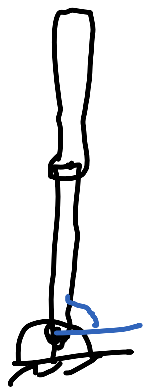
خطی سازی

$$
\begin{cases}
\dot{x}_1 = x_3 = f_1 \\
\dot{x}_2 = x_4 = f_2 \\
\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = M^{-1}(x_1, x_2)\left[\tau - V(x_1, \dots, x_4) - G(x_1, x_2)\right] \quad \begin{matrix} f_3 \\ f_4 \end{matrix}
\end{cases}
$$

$$
A = \frac{\partial F}{\partial x} \longrightarrow A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_4} \\ \vdots & \ddots & \\ \frac{\partial f_4}{\partial x_1} & & \frac{\partial f_4}{\partial x_4} \end{bmatrix}_{x^*, u^*}
$$

$$
B = \frac{\partial F}{\partial \tau} = \begin{bmatrix} \frac{\partial f_1}{\partial \tau_1} & \frac{\partial f_1}{\partial \tau_2} \\ \vdots & \\ \frac{\partial f_4}{\partial \tau_1} & \frac{\partial f_4}{\partial \tau_2} \end{bmatrix}
$$

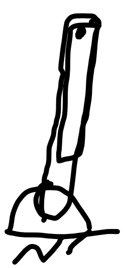
نقاط تعادل



1. $\theta_1 = 90, \quad \theta_2 = 0, \quad \tau_1 = \tau_2 = 0$

2. $\theta_1 = -90, \quad \theta_2 = 0, \quad \tau_1, \tau_2 = 0$

$$\dot{\theta}_1 = \dot{\theta}_2 = 0$$



3. $\theta_1 = 90, \quad \theta_2 = 180,$

4. $\theta_1 = -90, \quad \theta_2 = 180$

```
clc
clear
syms m1 m2 L1 L2 Lc1 Lc2 I1 I2 th1 th2 thd1 thd2 g tu1 tu2 D
m1=2;m2=1;L1=1;L2=0.5;I1=0.01;I2=0.005;%g=9.81;D=2;
Lc1=L1/2;Lc2=L2/2;
% Lc1=L1/2;Lc2=L2/2;I1=m1*L1^2/12;I2=m2*L2^2/12;
M=[m1*Lc1^2+m2*(L1^2+Lc2^2)+2*m2*L1*Lc2*cos(th2)+I1+I2  m2*Lc2^2+m2*L1*Lc2*cos(th2)+I2;
m2*Lc2^2+m2*L1*Lc2*cos(th2)+I2 m2*Lc2^2+I2];
V=[-2*m2*L1*Lc2*thd1*thd2*sin(th2)-m2*L1*Lc2*thd2^2*sin(th2); m2*L1*Lc2*thd1^2*sin(th2)];
G=[(m1*Lc1+m2*L1)*g*cos(th1)+m2*g*Lc2*cos(th1+th2); m2*g*Lc2*cos(th1+th2)];
Tu=[tu1;tu2];
qd=[thd1;thd2];
qdd=M^-1*(Tu-V-G-D*qd);
f1=thd1;
f2=thd2;
f3=qdd(1);
f4=qdd(2);
f=[f1;f2;f3;f4];
x=[th1;th2;thd1;thd2];
A=jacobian(f,x);
B=jacobian(f,Tu);
AA=subs(A,[th1;th2; thd1;thd2; tu1 ;tu2],[pi/2; 0;0;0;0;0]);
BB=subs(B,[th1;th2; thd1;thd2; tu1 ;tu2],[pi/2; 0;0;0;0;0]);
```

AA =

```
[                                   0,                                    0,                  1,                                  0]
[                                   0,                                    0,                  0,                                  1]
[            (3*g*(2*m1 + m2))/(L1*(4*m1 + 3*m2)),            -(9*g*m2)/(L1*(4*m1 + 3*m2)),         -(12*D)/(L1^2*(4*m1 + 3*m2)),
(6*D*(3*L1 + 2*L2))/(L1^2*L2*(4*m1 + 3*m2))]
[ -(3*g*(L1*m1 + 2*L2*m1 + L2*m2))/(L1*L2*(4*m1 + 3*m2)), (3*g*(2*L1*m1 + 6*L1*m2 + 3*L2*m2))/(L1*L2*(4*m1 + 3*m2)), (6*D*(3*L1 +
2*L2))/(L1^2*L2*(4*m1 + 3*m2)), -(12*D*(L1^2*m1 + 3*L1^2*m2 + L2^2*m2 + 3*L1*L2*m2))/(L1^2*L2^2*m2*(4*m1 + 3*m2))]
```

BB =

```
[                                0,                                       0]
[                                0,                                       0]
[             12/(L1^2*(4*m1 + 3*m2)),              -(6*(3*L1 + 2*L2))/(L1^2*L2*(4*m1 + 3*m2))]
[ -(6*(3*L1 + 2*L2))/(L1^2*L2*(4*m1 + 3*m2)), (12*(L1^2*m1 + 3*L1^2*m2 + L2^2*m2 +
3*L1*L2*m2))/(L1^2*L2^2*m2*(4*m1 + 3*m2))]
```

AA =

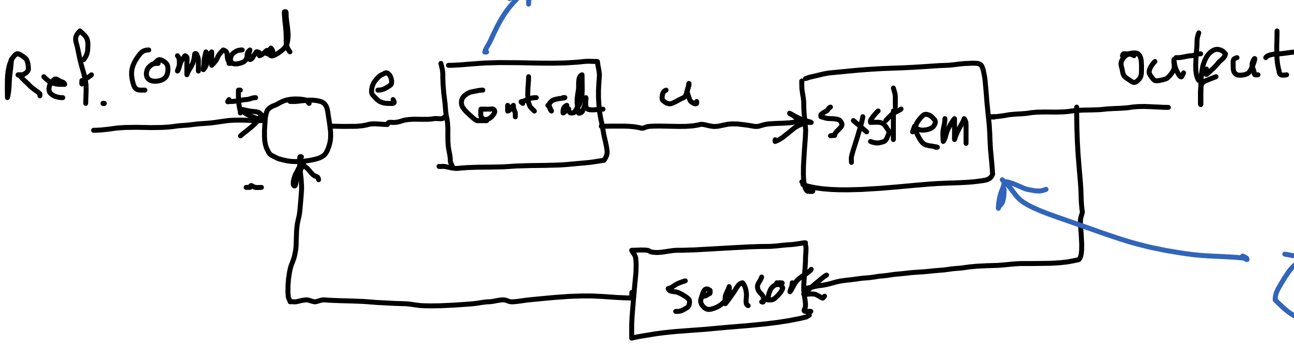
```
[            0,             0,             1,             0]
[            0,             0,             0,             1]
[  (2900*g)/1577, -(2500*g)/1577, -(2700*D)/1577,  (12700*D)/1577]
[ -(7800*g)/1577, (17600*g)/1577, (12700*D)/1577, -(83100*D)/1577]
```

BB =

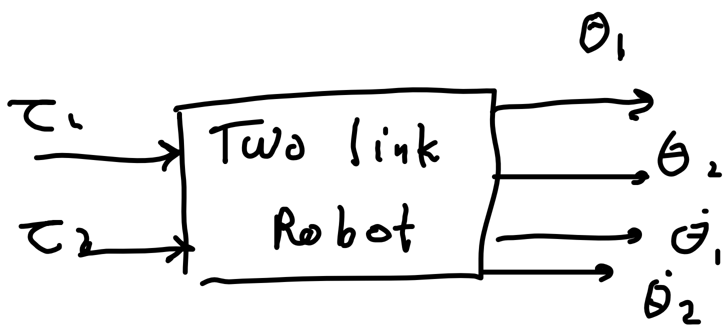
```
[          0,           0]
[          0,           0]
[   2700/1577, -12700/1577]
[ -12700/1577,  83100/1577]
```

شبیه سازی ربات

State feedback



معادلات دینامیکی استخراج شد. استفاده می کنیم



$$\tau_u \longrightarrow \boxed{M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau - D\dot{q}} \longrightarrow q$$

حل معادلات دینامیک مستقیم ، معادل حل یک دستگاه معادله ode

معادلات غیرخطی هستند ، استفاده از روشهای عددی برای حل معادله،

در MATLAB

کدنویسی در صفحه M file ← ode45 ✓

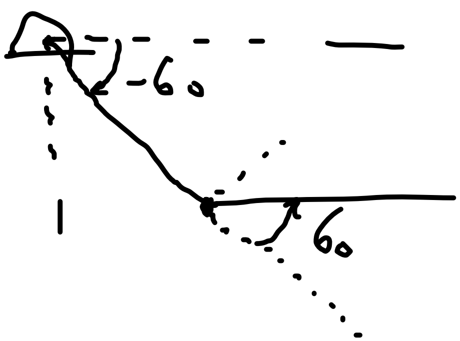
شبیه سازی در محیط simulink ✓

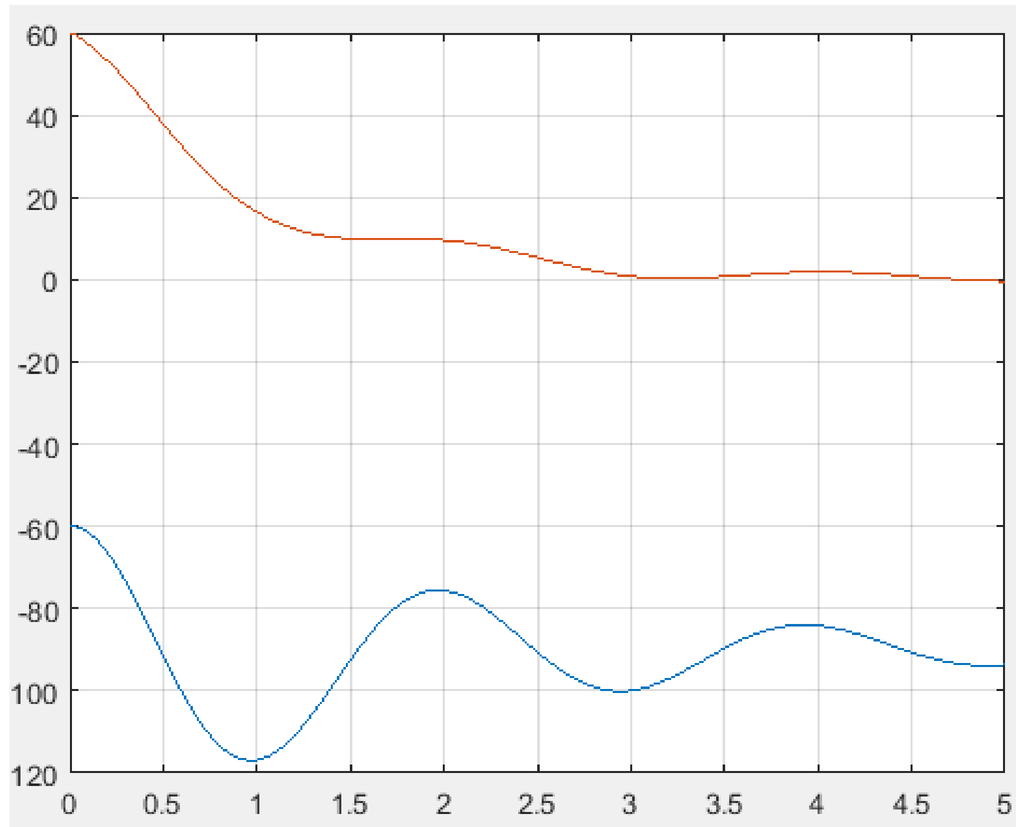
مدلسازی مستقیم

MATLAB fun

simmechanics

q1





طراحی کنترلر فیدبک حالت به روش LQR

$$A, \; B, \; R, \; Q$$

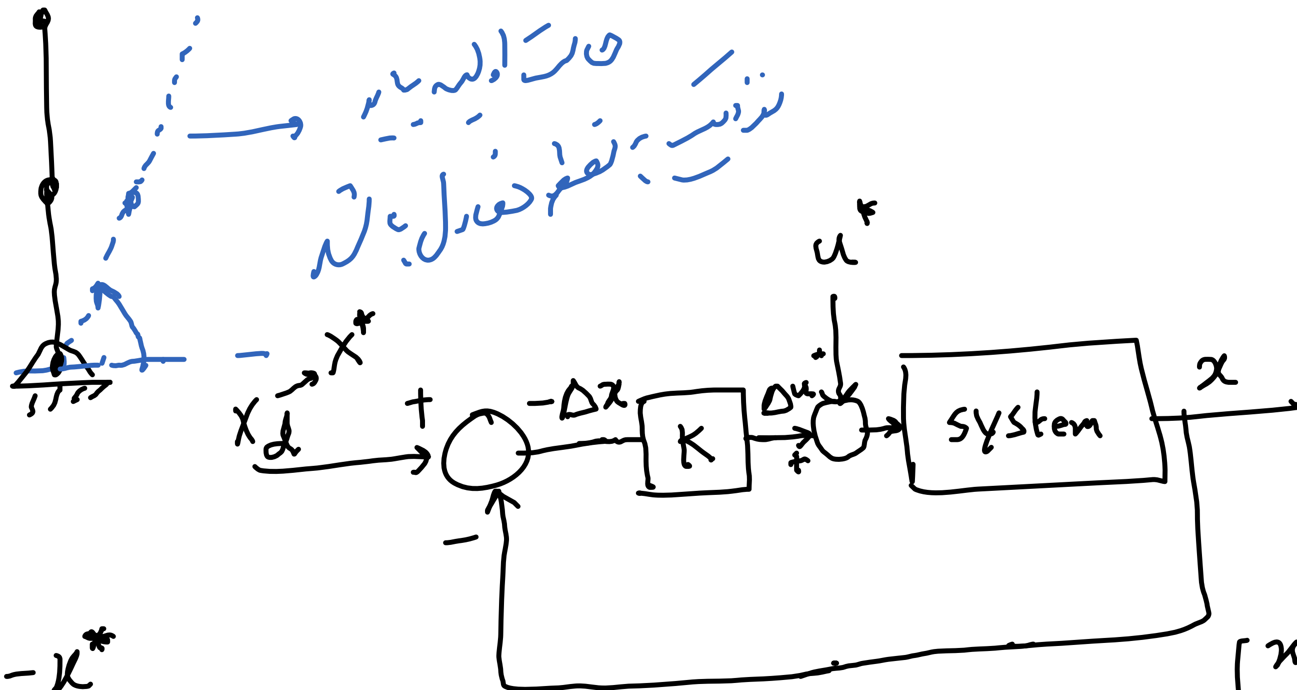
$$R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} , \qquad Q = \begin{bmatrix} q_1 & & & \\ & q_2 & & \\ & & q_3 & \\ & & & q_4 \end{bmatrix}$$

ضریب [illegible] $q_1 \longrightarrow x_1 , \; \theta_1$

$q_2 \longrightarrow x_2 , \; \theta_2$

$q_3 \longrightarrow x_3 , \; \dot{\theta}_1$

$q_4 \longrightarrow x_4 , \; \dot{\theta}_2$



x_{de}

$$\Delta x = x - x^*$$

$$u = \tau = \tau^* + \Delta\tau = \tau^* - K\Delta x \quad , \quad \Delta x = \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \\ x_3 - x_3^* \\ x_4 - x_4^* \end{bmatrix}$$

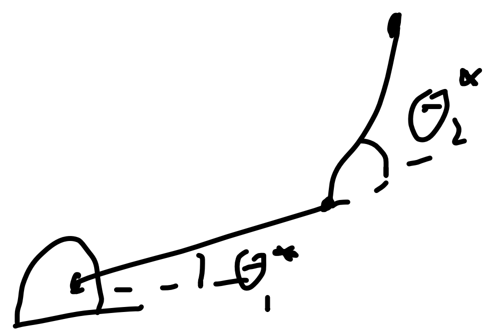
خطی سازی حول یک نقطه دلخواه!

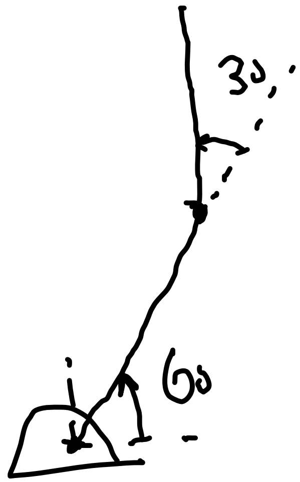
$$M\underbrace{\ddot{q}}_{0} + \underbrace{V(q,\dot{q})}_{0} + G(q) + \underbrace{D\dot{q}}_{0} = \tau$$

در حالت تعادل باشیم

$$\dot{q} = \ddot{q} = 0$$

$$\tau^* = G(q^*)$$





$$\tau_2^* = 0$$

$$\tau_1^* \neq 0$$