کنترل اتوماتیک

تحلیل پاسخ فرکانسی
دیاگرام بود، بخش اول

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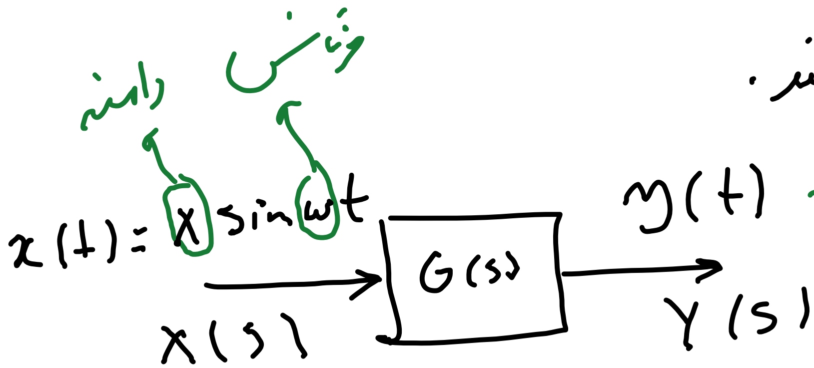
تحلیل پاسخ فرکانسی: Frequency Response Analysis

منظور از پاسخ فرکانسی، پاسخ حالت ماندگار سیستم به ورودی سینوسی می‌باشد.

در روش‌های پاسخ فرکانسی، فرکانس سیگنال ورودی را در یک گستره فرکانسی خاص تغییر داده و پاسخ حاصل را بررسی می‌کنند.



$$\Rightarrow y_{ss} = y(t) \quad t \to \infty$$

تابع تبدیل $G(s)$ را در حالت کلی به صورت زیر در نظر بگیرید:

$$G(s) = \frac{P(s)}{(s+s_1)(s+s_2)\cdots(s+s_n)}$$

$$X(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

$X = cte$

$$x(t) = X \sin \omega t \longrightarrow X(s) = \frac{X\omega}{s^2+\omega^2}$$

$$Y(s) = G(s)X(s) = \frac{P(s)}{(s+s_1)(s+s_2)\cdots(s+s_n)} \times \frac{X\omega}{s^2+\omega^2}$$

$$= \frac{a}{s+j\omega} + \frac{\bar{a}}{s-j\omega} + \frac{b_1}{s+s_1} + \frac{b_2}{s+s_2} + \cdots + \frac{b_n}{s+s_n}$$

بسط به کسرهای جزئی

با گرفتن عکس تبدیل لاپلاس

$$y(t) = a e^{-j\omega t} + \bar{a} e^{j\omega t} + \cancel{b_1 e^{-s_1 t}} + \cdots + \cancel{b_n e^{-s_n t}}$$

$$t \longrightarrow \infty$$

$$y_{ss} = a e^{-j\omega t} + \bar{a} e^{j\omega t}$$

$$a = \lim_{s \to -j\omega} \left[G(s) \frac{\omega X}{s^2+\omega^2} (\cancel{s+j\omega}) \right] = -\frac{X G(-j\omega)}{2j}$$

$$s^2+\omega^2 \longrightarrow (\cancel{s+j\omega})(s-j\omega)$$

$$\bar{a} = \lim_{s \to j\omega} \left[G(s) \frac{\omega X}{s^2+\omega^2} (s-j\omega) \right] = \frac{X G(j\omega)}{2j}$$

از طرف دیگر $G(s)$ یک کمیت مختلط است

$$G(j\omega) = |G(j\omega)| e^{j\phi} \quad \text{و} \quad \phi = \angle G(j\omega)$$

$$G(-j\omega) = |G(j\omega)| e^{-j\phi}$$

$$\Rightarrow a = -\frac{X\,G(-j\omega)}{2j} = -\frac{X\,|G(j\omega)|e^{-j\phi}}{2j}$$

$$\Rightarrow \bar{a} = \frac{X\,G(j\omega)}{2j} = \frac{X\,|G(j\omega)|e^{j\phi}}{2j}$$

$$y_{ss} = a e^{-j\omega t} + \bar{a} e^{j\omega t} =$$

$$-\frac{X|G(j\omega)|}{2j} e^{-j(\omega t+\varphi)} + \frac{X|G(j\omega)| e^{j(\omega t+\varphi)}}{2j}$$

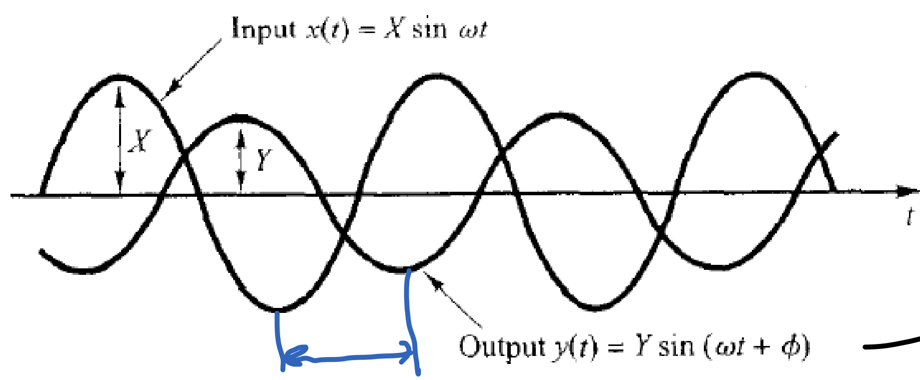
$$= X|G(j\omega)| \left(\frac{e^{j(\omega t+\varphi)} - e^{-j(\omega t+\varphi)}}{2j} \right)$$

$$= X|G(j\omega)| \sin(\omega t+\varphi)$$

$$x(t) = X \sin \omega t \longrightarrow \boxed{G(s)} \longrightarrow y_{ss} = X\,|G(j\omega)|\,\sin(\omega t + \varphi)$$

$$\varphi = \angle G(j\omega)$$

- دامنه خروجی به اندازه $|G(j\omega)|$ تغییر می یابد
- خروجی به اندازه $\varphi = \angle G(j\omega)$ نسبت به ورودی اختلاف فاز دارد.
- فرکانس نوسانات ورودی و خروجی تغییر نمی کند

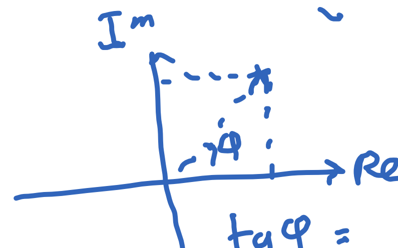


$$Y = X\,|G(j\omega)|$$

مثال: خروجی حالت ماندگار سیستم زیر را به ازای ورودی سینوسی محاسبه کنید

$$x\sin\omega t \longrightarrow \boxed{\frac{K}{Ts+1}} \longrightarrow$$

$$y_{ss} = X\,|G(j\omega)|\,\sin(\omega t+\varphi)$$



$$tg\,\varphi = \frac{Im}{Re}$$

$$G(s) = \frac{K}{Ts+1} \Rightarrow G(j\omega) = \frac{K}{T\omega j+1}$$

$$|G(j\omega)| = \frac{K}{\sqrt{T^2\omega^2+1}}$$

$$\varphi = \angle G(j\omega) = \cancel{\angle K} - \angle(T\omega j+1) = -tg^{-1}\frac{T\omega}{1} = -tg^{-1}T\omega$$

$$\Rightarrow y_{ss} = \frac{KX}{\sqrt{T^2\omega^2+1}}\sin(\omega t - tg^{-1}T\omega)$$

$$y_{ss} = \boxed{\frac{KX}{\sqrt{T^2\omega^2+1}}} \sin(\omega t - tg^{-1}T\omega)$$

فرض

$K=1,\ X=1,$

$T=1$

$\omega = 0.1 \longrightarrow$ دامنه خروجی $\frac{1}{\sqrt{1.01}} \simeq 1 \quad , \quad \varphi \simeq 0$

$\omega = 1 \longrightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.7 \quad , \quad \varphi = -45^\circ$

$\omega = 10 \xrightarrow{\text{دامنه خروجی}} \frac{1}{\sqrt{101}} \simeq \frac{1}{10} \quad , \quad \varphi \simeq -90$

$\sin 10t$

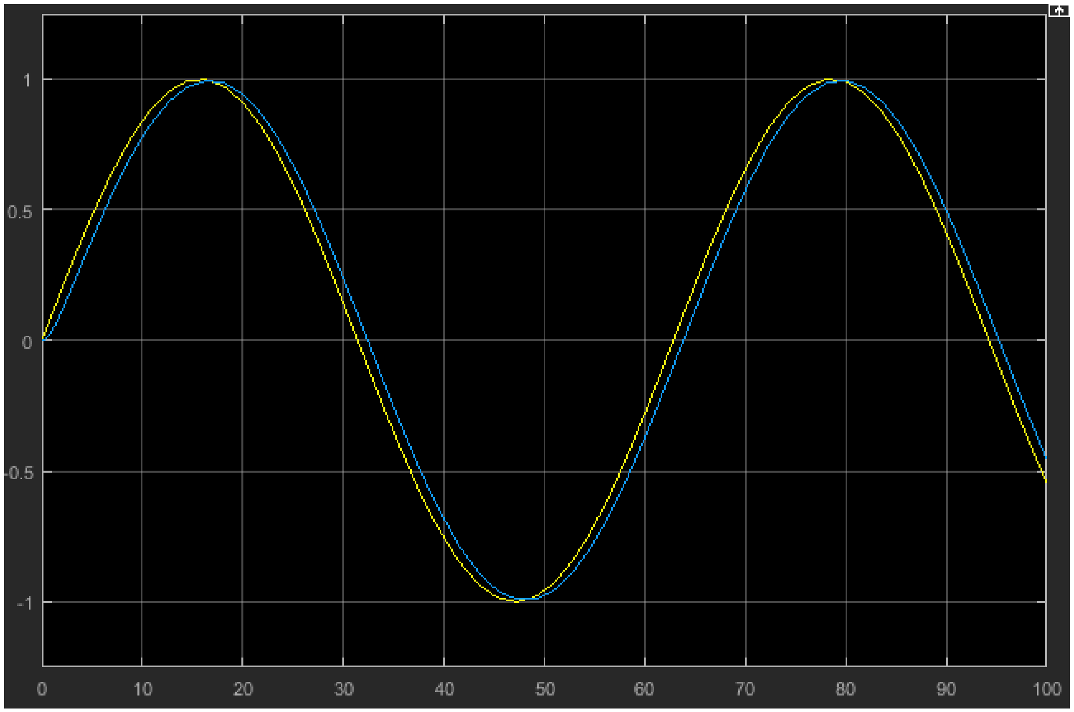
$\omega = 10 \Rightarrow y_{ss} = \frac{1}{10}\sin(10t - 90) \quad , \quad \omega = 0.1 \rightarrow y_{ss} = 1\sin(0.1t)$

$$x(t) = \sin(0.1t) \longrightarrow y_{ss} = \sin(0.1t)$$

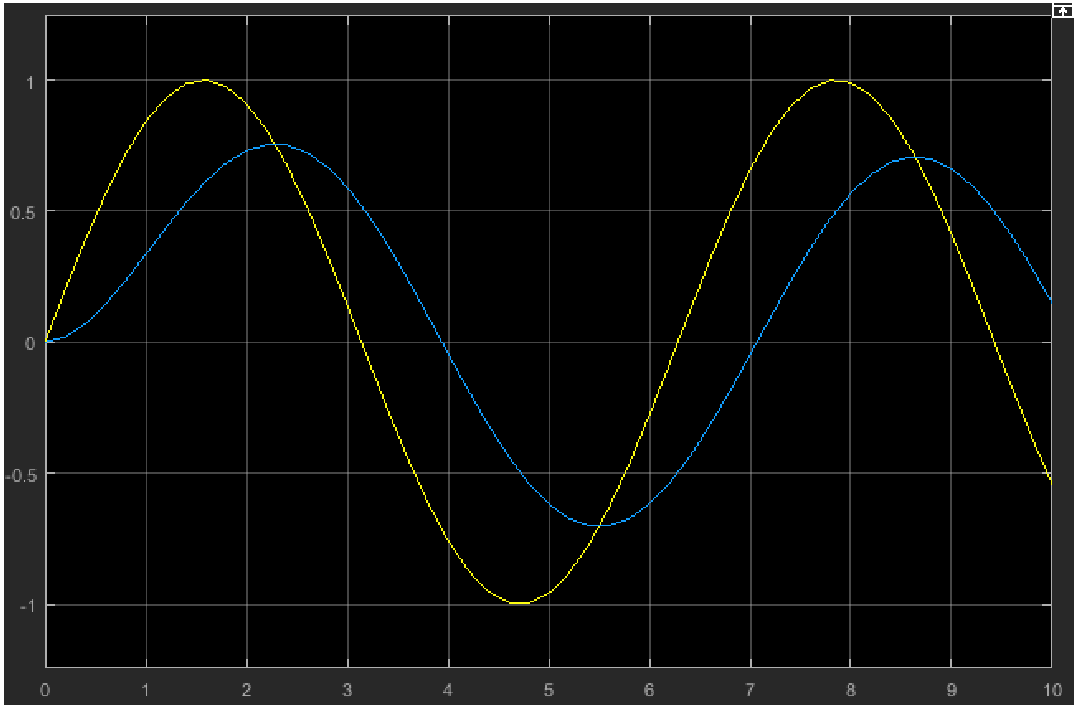
$$x(t) = \sin(t) \longrightarrow y_{ss} = \frac{\sqrt{2}}{2}\sin(t - 45^\circ)$$

$$x(t) = \sin(10t) \longrightarrow y_{ss} = \frac{1}{10}\sin(10t - 90^\circ)$$

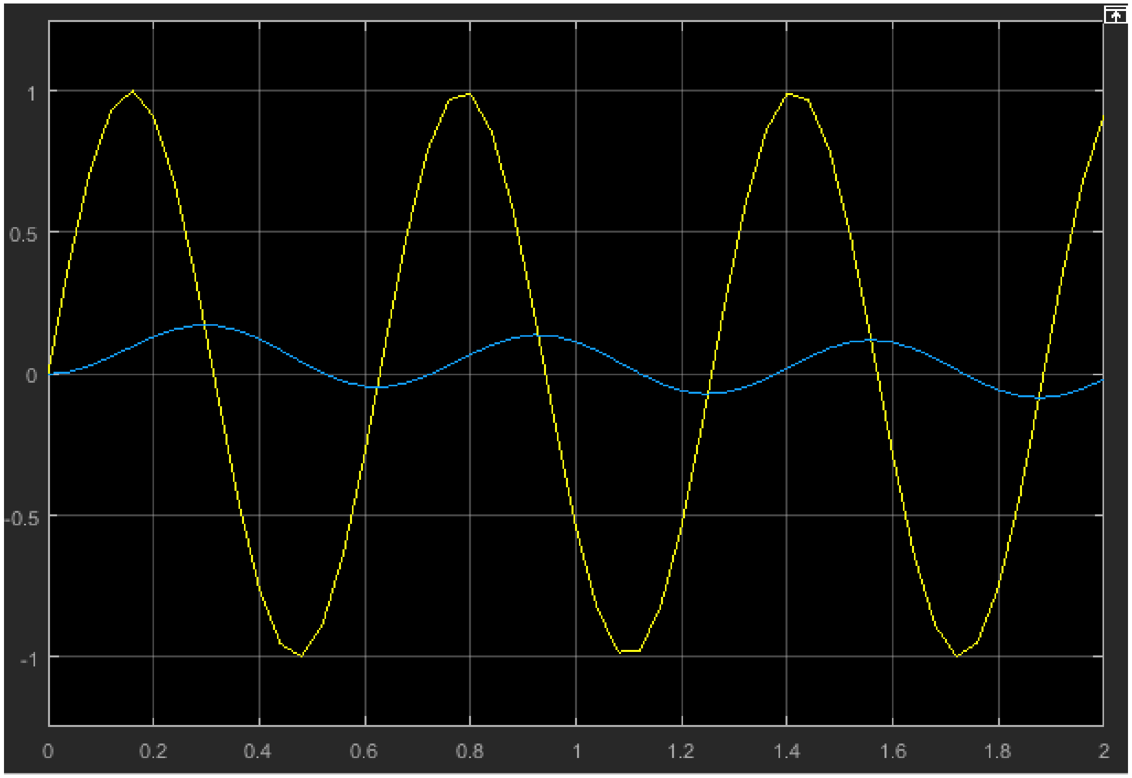
$$X(s) \longrightarrow \boxed{\frac{1}{s+1}} \longrightarrow Y_{ss}$$



$$x(t) = \sin(0.1t) \longrightarrow y_{ss} = \sin(0.1t)$$



$$x(t) = \sin(t) \longrightarrow y_{ss} = \frac{\sqrt{2}}{2}\sin(t - 45^{\circ})$$



$$x(t) = \sin(10t) \longrightarrow y_{ss} = \frac{1}{10}\sin(10t - 90^\circ)$$

$$x\sin\omega t \longrightarrow \boxed{G(s)} \longrightarrow x\,|G(j\omega)|\,\sin(\omega t+\varphi)$$

$$\varphi = \angle G(j\omega)$$

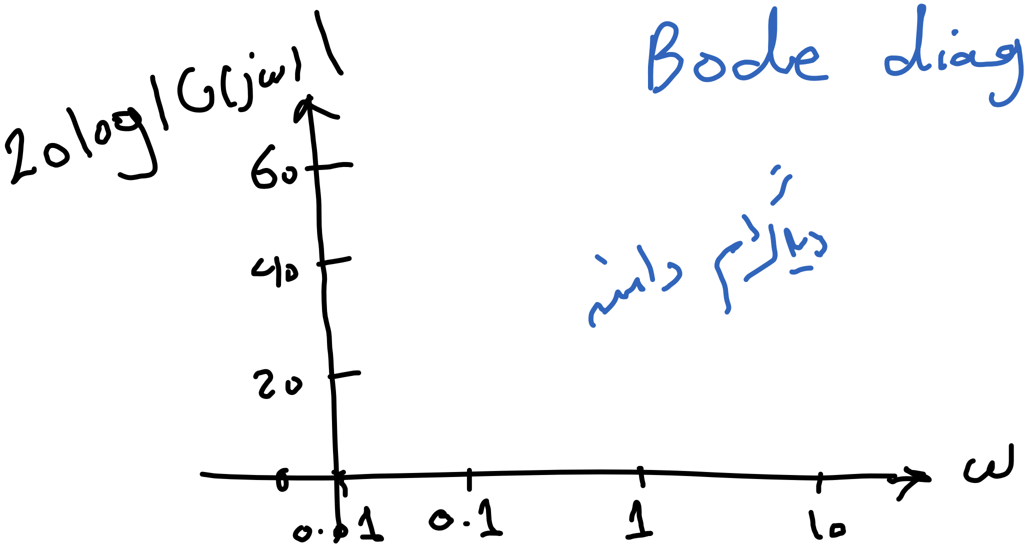
جهت تحلیل پاسخ فرکانسی باید $|G(j\omega)|$ و $\varphi = \angle G(j\omega)$ را به ازای مقادیر مختلف ω مورد بررسی قرار دهیم. روشهای مختلفی برای این کار وجود دارد. یکی از پرکاربردترین این روشها دیاگرام بودی می‌باشد

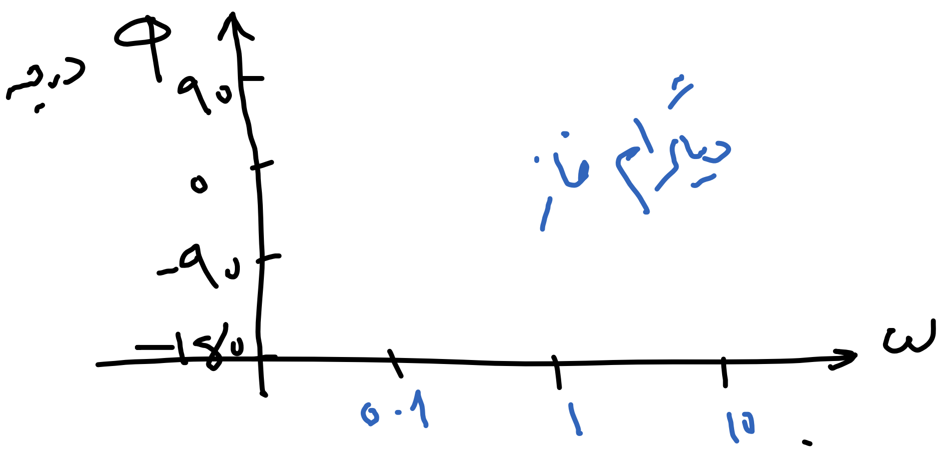
Bode diagram

Bode diagram

دیاگرام بود

$$G(s) = \frac{K(T_1 s+1)(T_2 s+1)}{(T_3 s+1)(T_4 s+1)}$$





$$G(s) = \frac{K(T_1 s+1)(T_2 s+1)}{(T_3 s+1)(T_4 s+1)} ,$$

$$|G(j\omega)| = \frac{|K||(T_1 \omega j + 1)||(T_2 \omega j + 1)|}{|(T_3 \omega j + 1)||(T_4 \omega j + 1)|}$$

$$\log|G(j\omega)| = \log|K| + \log|T_1 \omega j + 1| + \log|T_2 \omega j + 1| - \log|T_3 \omega j + 1| - \log|T_4 \omega j + 1|$$

$$\varphi = \angle G(j\omega) = \angle K + \angle T_1 \omega j + 1 + \angle T_2 \omega j + 1 - \angle T_3 \omega j + 1 - \angle T_4 \omega j + 1$$

عوامل پایه‌ای

Basic Factors of $G(j\omega)H(j\omega)$. As stated earlier, the main advantage in using the logarithmic plot is the relative ease of plotting frequency-response curves. The basic factors that very frequently occur in an arbitrary transfer function $G(j\omega)H(j\omega)$ are

1. Gain K
2. Integral and derivative factors $(j\omega)^{\mp 1}$
3. First-order factors $(1 + j\omega T)^{\mp 1}$
4. Quadratic factors $[1 + 2\zeta(j\omega/\omega_n) + (j\omega/\omega_n)^2]^{\mp 1}$

$$G(s)=\frac{\overset{1}{K}\,\overset{3}{(s+1)}}{\underset{2}{s^2}\,\underset{4}{(s^2+s+1)}\,\underset{3}{(s+3)}}$$

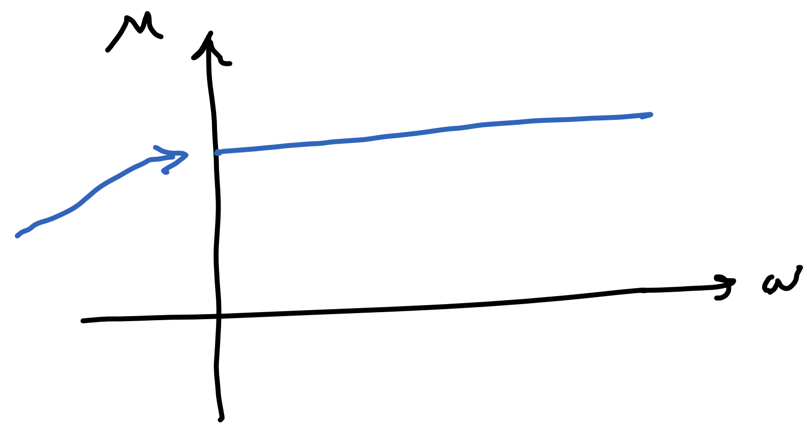
۱- عامل پایایی ثابت

$$G(s) = K$$

$$G(j\omega) = K ,$$

$$M = 20\log|G(j\omega)| = 20\log K$$

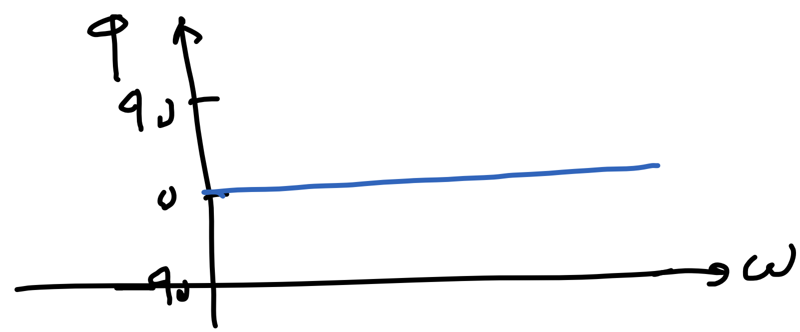
$$\varphi = \angle G(j\omega) = \angle K = 0$$



$K=10 \rightarrow M = 20$

$K=1 \rightarrow M=0$

$K=0.1 \rightarrow M=-20$



۲ - عامل مشتق گیر یا انتگرال گیر

عامل انتگرالی

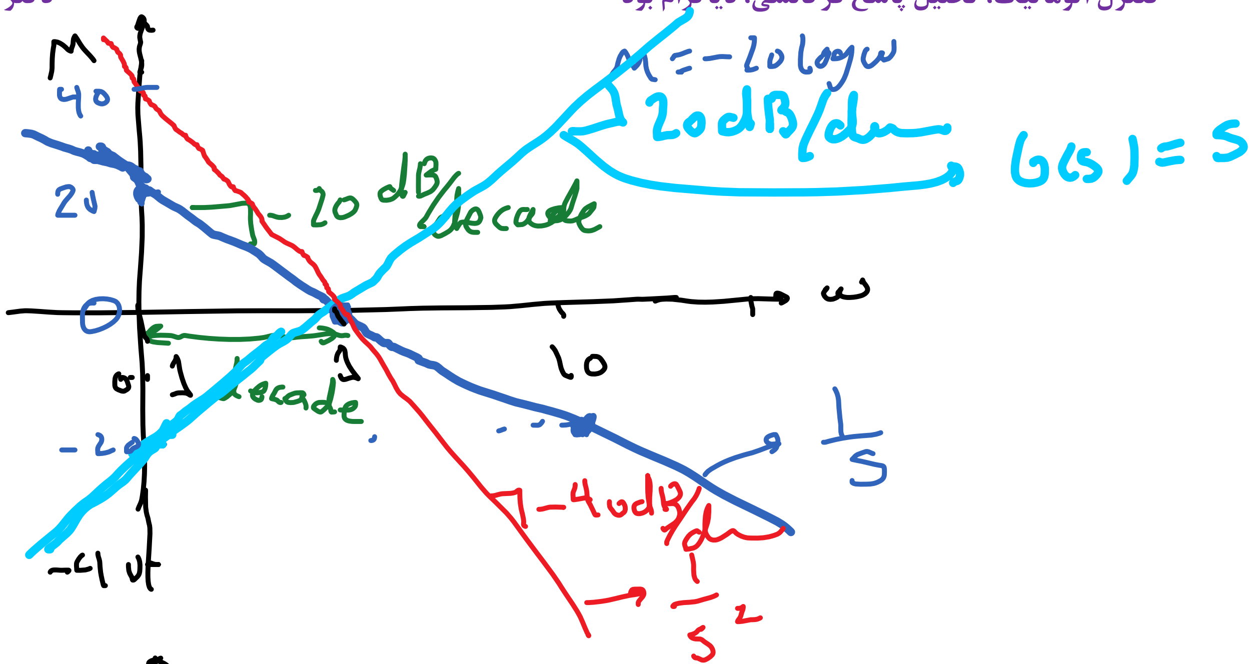
$$G(s) = \frac{1}{s} \longrightarrow G(j\omega) = \frac{1}{j\omega}$$

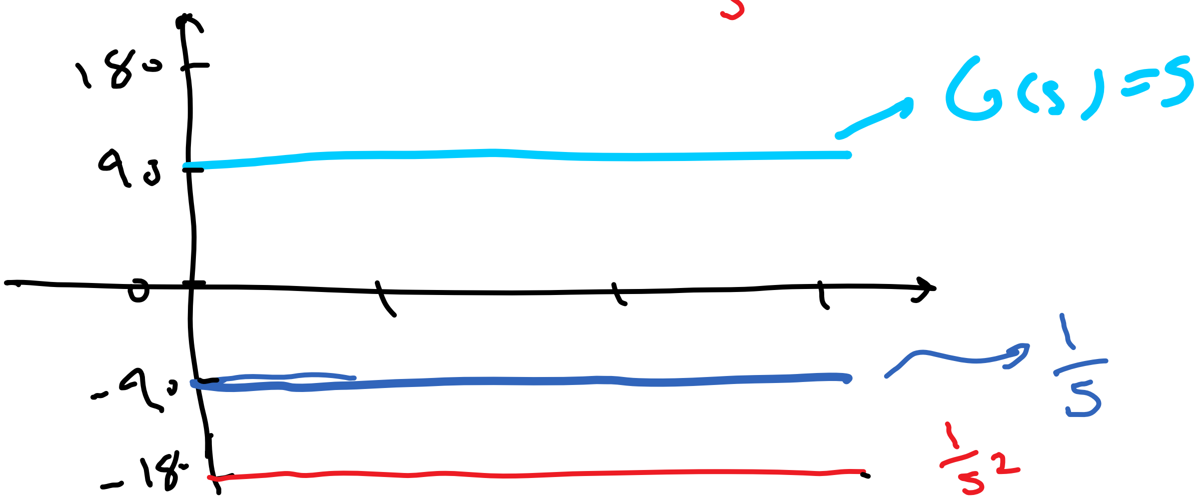
$$M = 20 \log \left| \frac{1}{j\omega} \right| = 20 \log \frac{1}{\omega} = -20 \log \omega$$

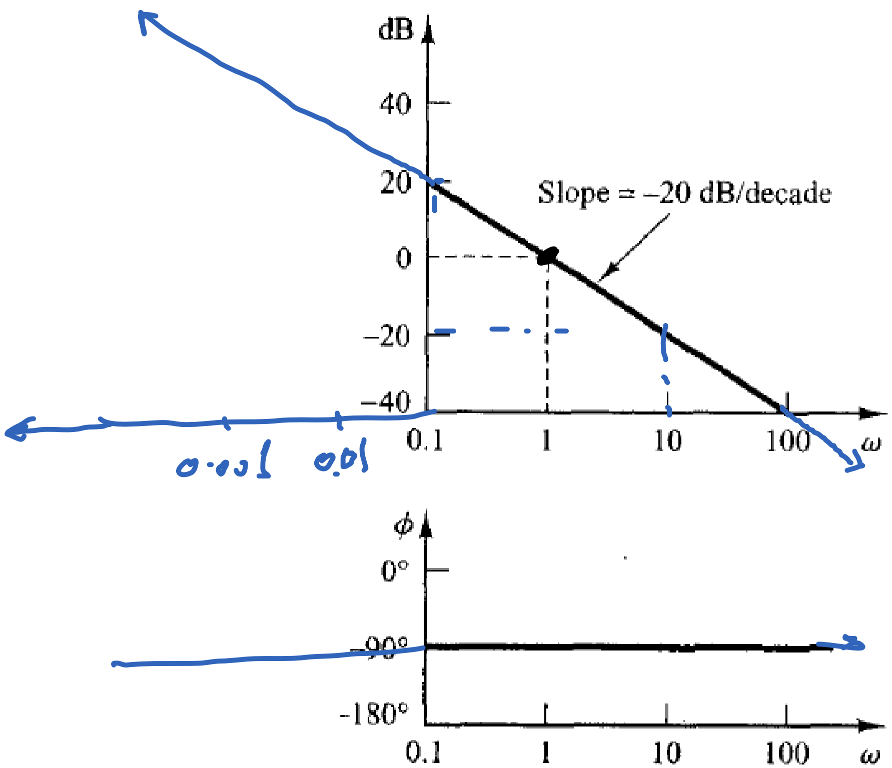
$$\varphi = \angle 1 - \angle j\omega = 0 - 90 = -90$$

مشتق $G(s) = s \longrightarrow M = 20 \log \omega$

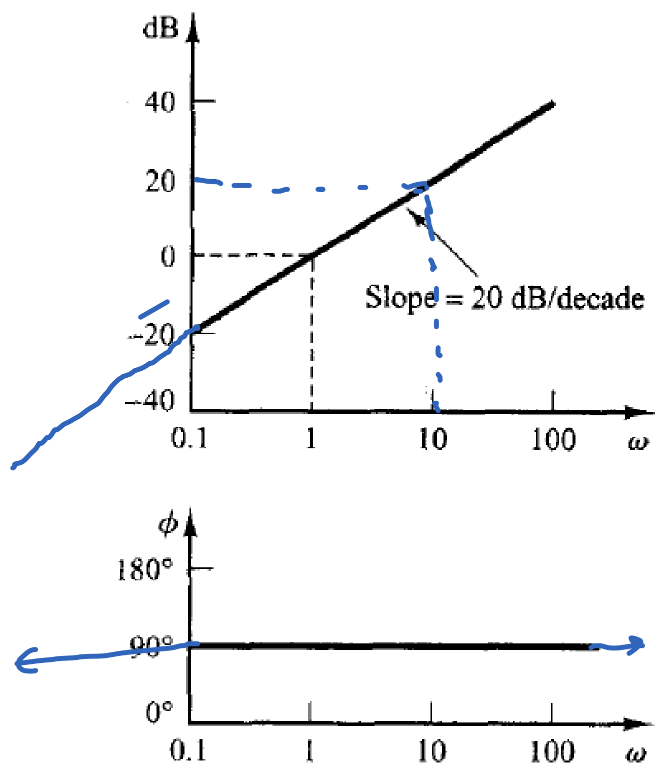
$$\varphi = 90$$


M
40
20
0
-20
-40
0.1
1
10
ω
M=-20 log ω
20dB/dec
G(s) = s
-20 dB/decade
decade
-40dB/dec
1/s
1/s²


180
90
0
-90
-180
G(s)=s
1/s
1/s²



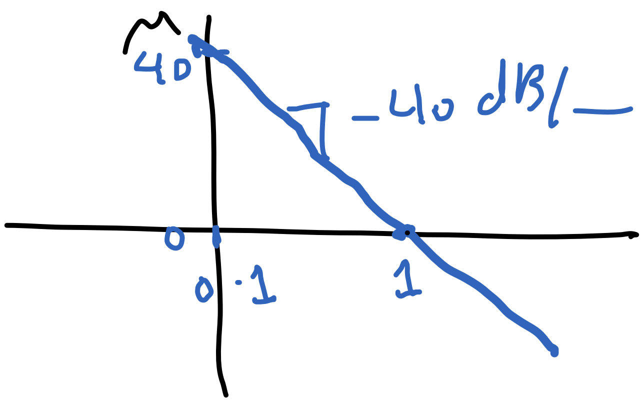
Bode diagram of
$G(j\omega) = 1/j\omega$

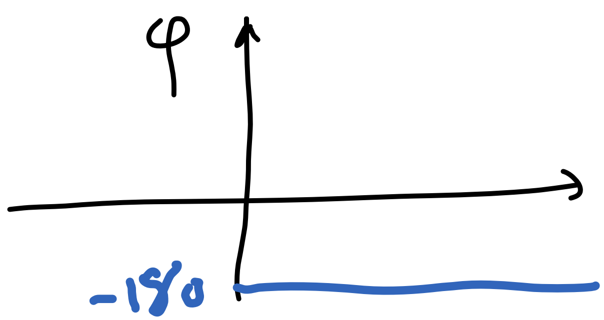


Bode diagram of
$G(j\omega) = j\omega$

$$G(s) = \frac{1}{s^2} = \frac{1}{s}\,\frac{1}{s}$$

$$20\log|G(j\omega)| = 20\log\left|\frac{1}{j\omega}\right| + 20\log\left|\frac{1}{j\omega}\right|$$



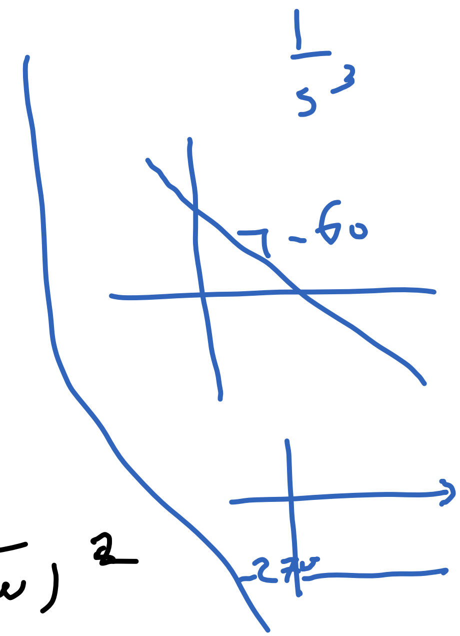


$$G(s) = \frac{1}{s^2}$$

$$G(j\omega) = \frac{1}{(j\omega)^2}$$

$$M = 20\log\left|\frac{1}{(j\omega)^2}\right| = -40\log\omega$$

$$\varphi = 0 - \angle j\omega - \angle j\omega = -90 - 90 = -180$$



۳- عامل پایه‌ای مرتبه اول

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{T\omega j+1} \rightarrow M = -20 \log |T\omega j+1|$$

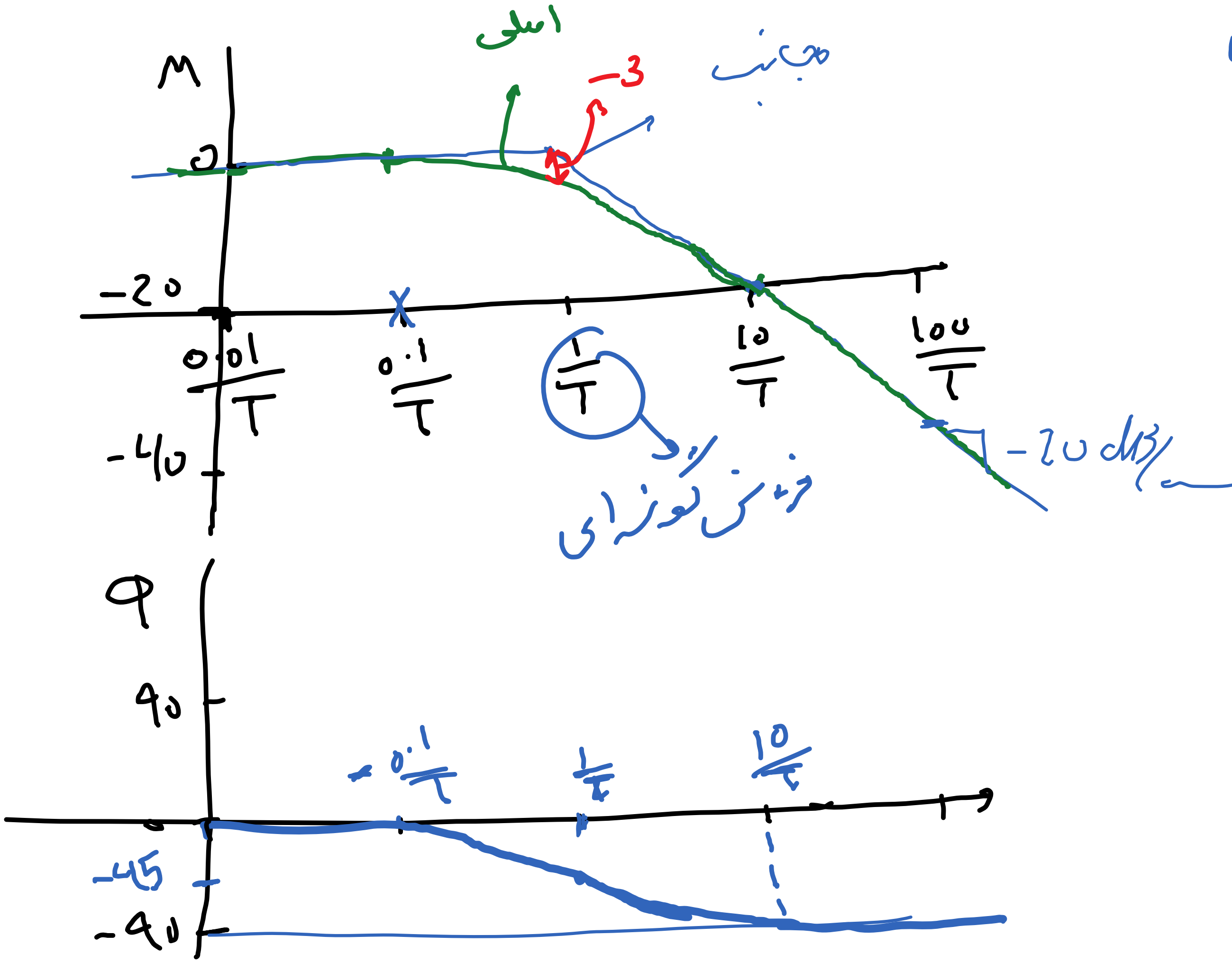
$$\Rightarrow M = -20 \log \sqrt{T^2\omega^2+1} \Rightarrow M \begin{cases} 0 & \omega \ll \frac{1}{T} \\ -20 \log T\omega & \omega \gg \frac{1}{T} \end{cases}$$

$$\varphi = \angle 1 - \angle (T\omega j+1) = 0 - tg^{-1} T\omega$$

$$\rightarrow \begin{cases} 0 & \omega \ll \frac{1}{T} \\ -90 & \omega \gg \frac{1}{T} \end{cases}$$

$$\omega = \frac{10}{T}$$

$$-20 \log \cancel{T} \frac{10}{\cancel{T}} = -20$$



$$\omega = \frac{10}{T} \rightarrow M = -20$$

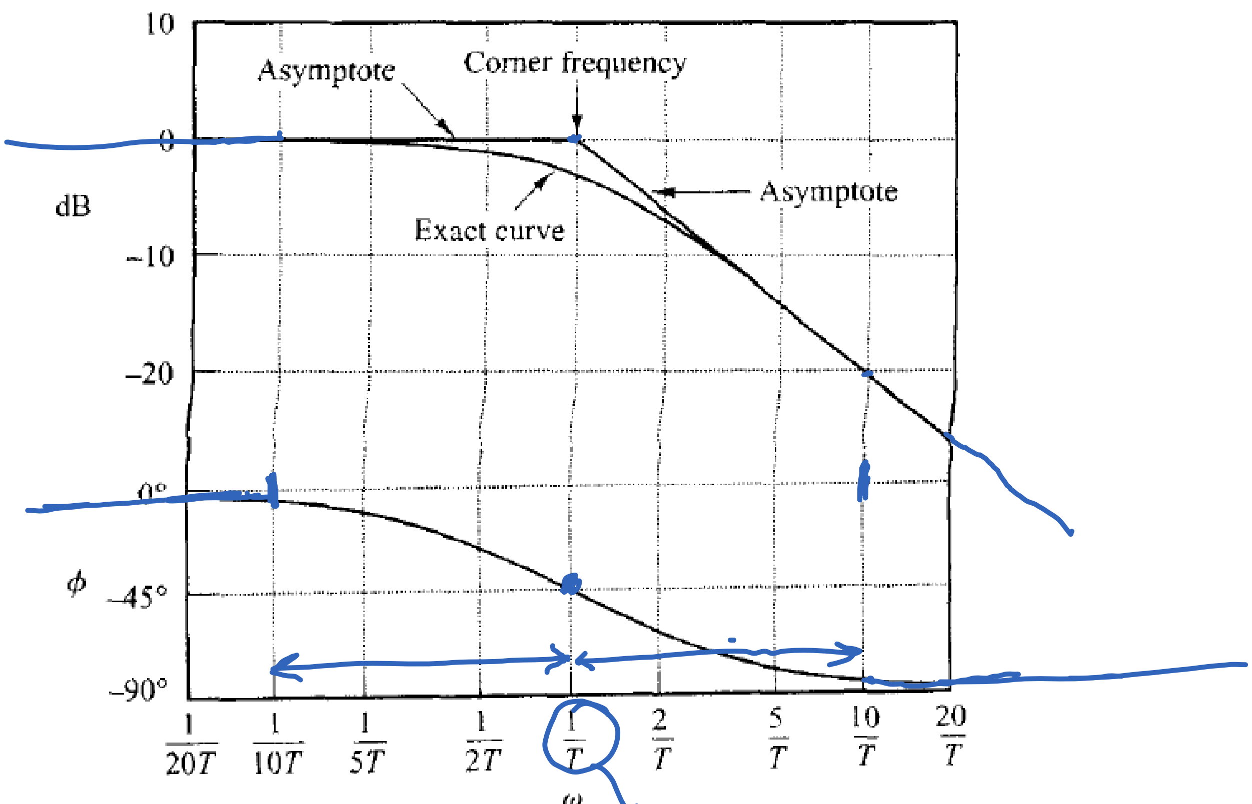
$$M = -20 \log \sqrt{T^2\omega^2 + 1}$$

$$\omega = \frac{1}{T} \Rightarrow M = -20 \log \sqrt{2}$$

$$\rightarrow M = -3$$

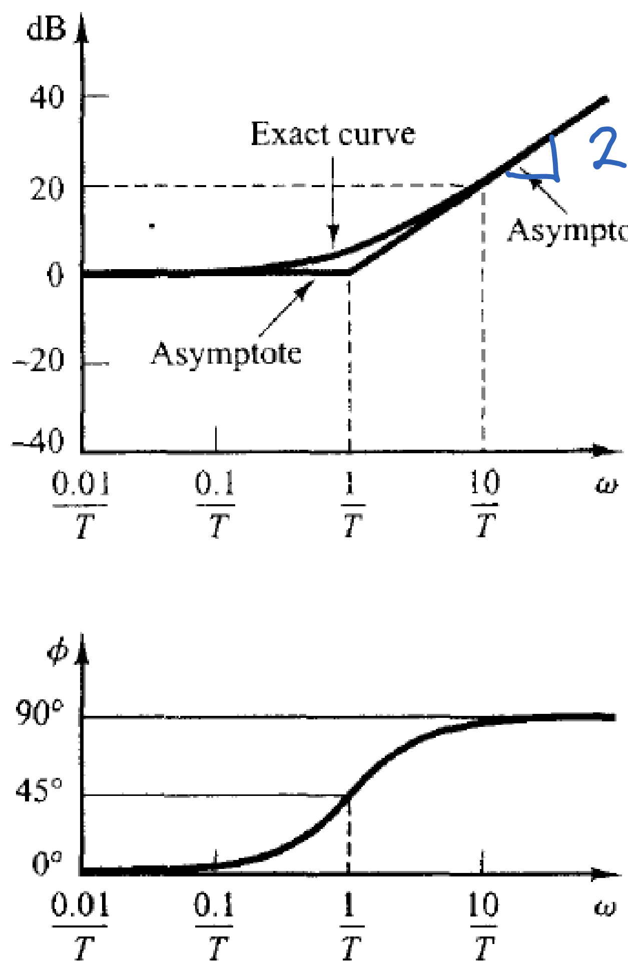
$$\omega = \frac{1}{T} \rightarrow \varphi = 45^{-}$$

$$\frac{1}{Ts+1}$$



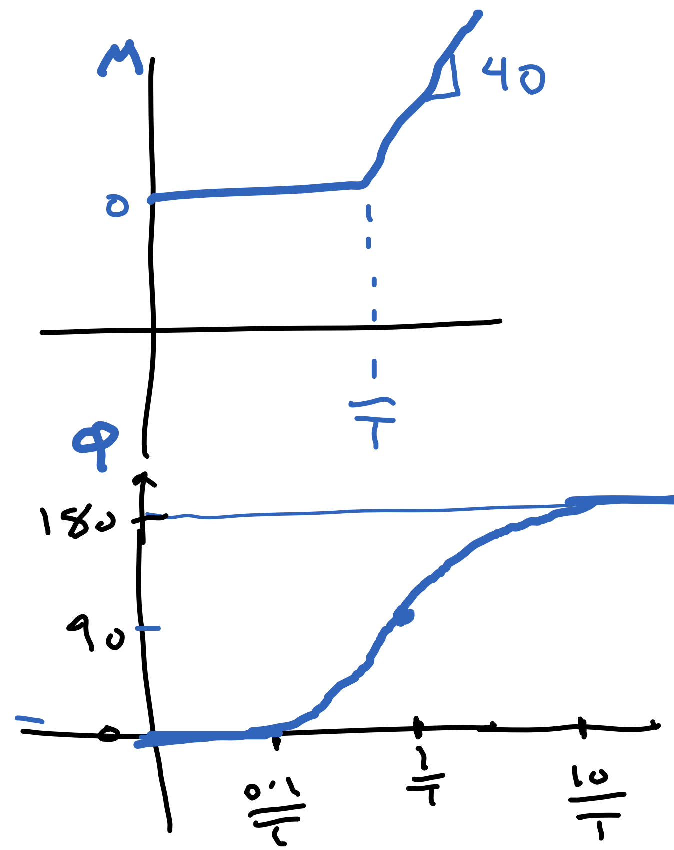
فرکانس گوشه‌ای

$G(s) = Ts+1$



20 dB/dec

$(Ts+1)^2$



٤ - عامل پایه‌ی مرتبه دوم

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta s + \omega_n^2}$$

$$G(s) = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1}$$

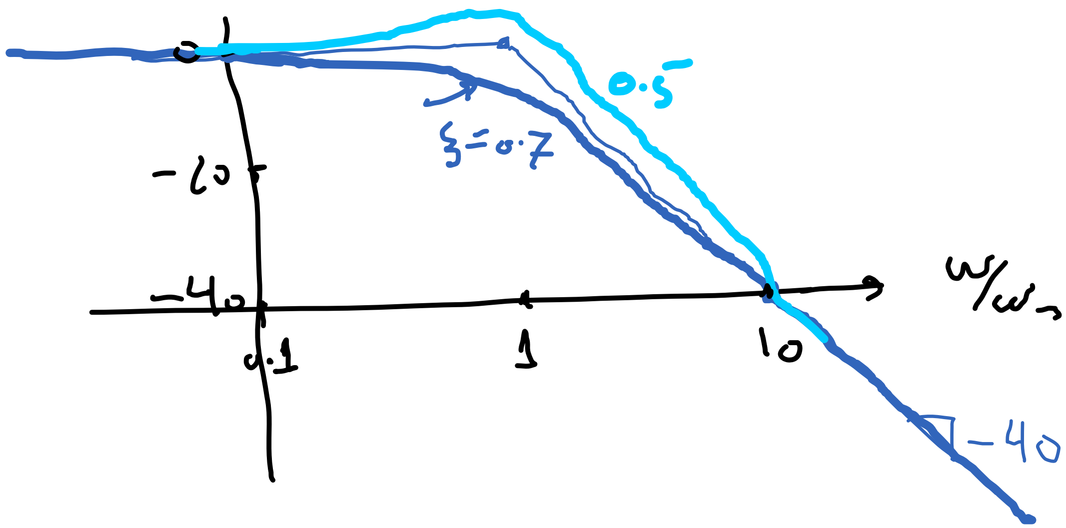
$$G(j\omega) = \frac{1}{\left(\frac{j\omega}{\omega_n}\right)^2 + \frac{2\zeta j\omega}{\omega_n} + 1} = \frac{1}{1 - \frac{\omega^2}{\omega_n} + \frac{2\zeta\omega}{\omega_n} j}$$

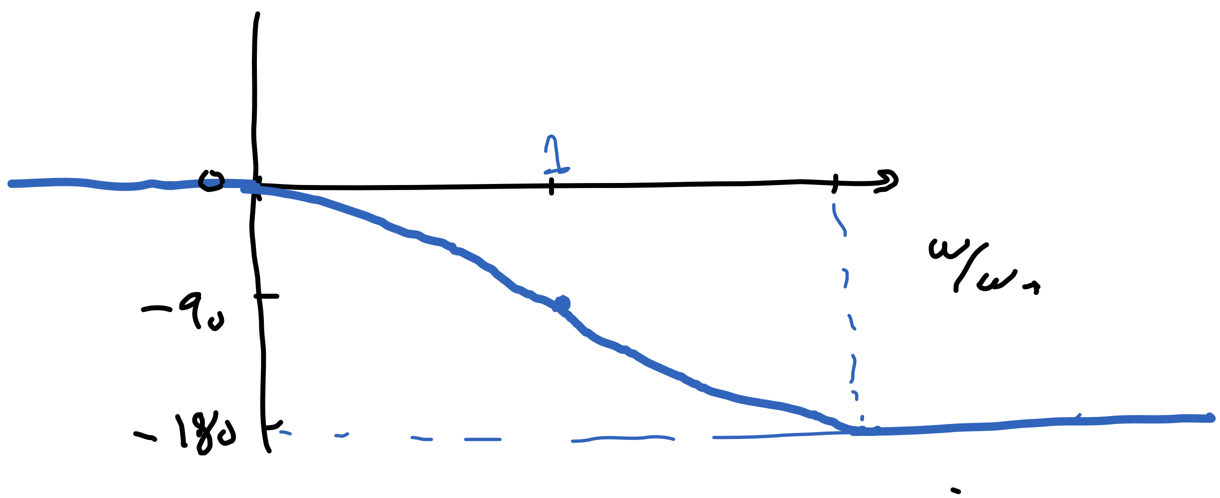
$$G(j\omega)=\frac{1}{1-\frac{\omega^2}{\omega_n^2}+\frac{2\xi\omega}{\omega_n}j}$$

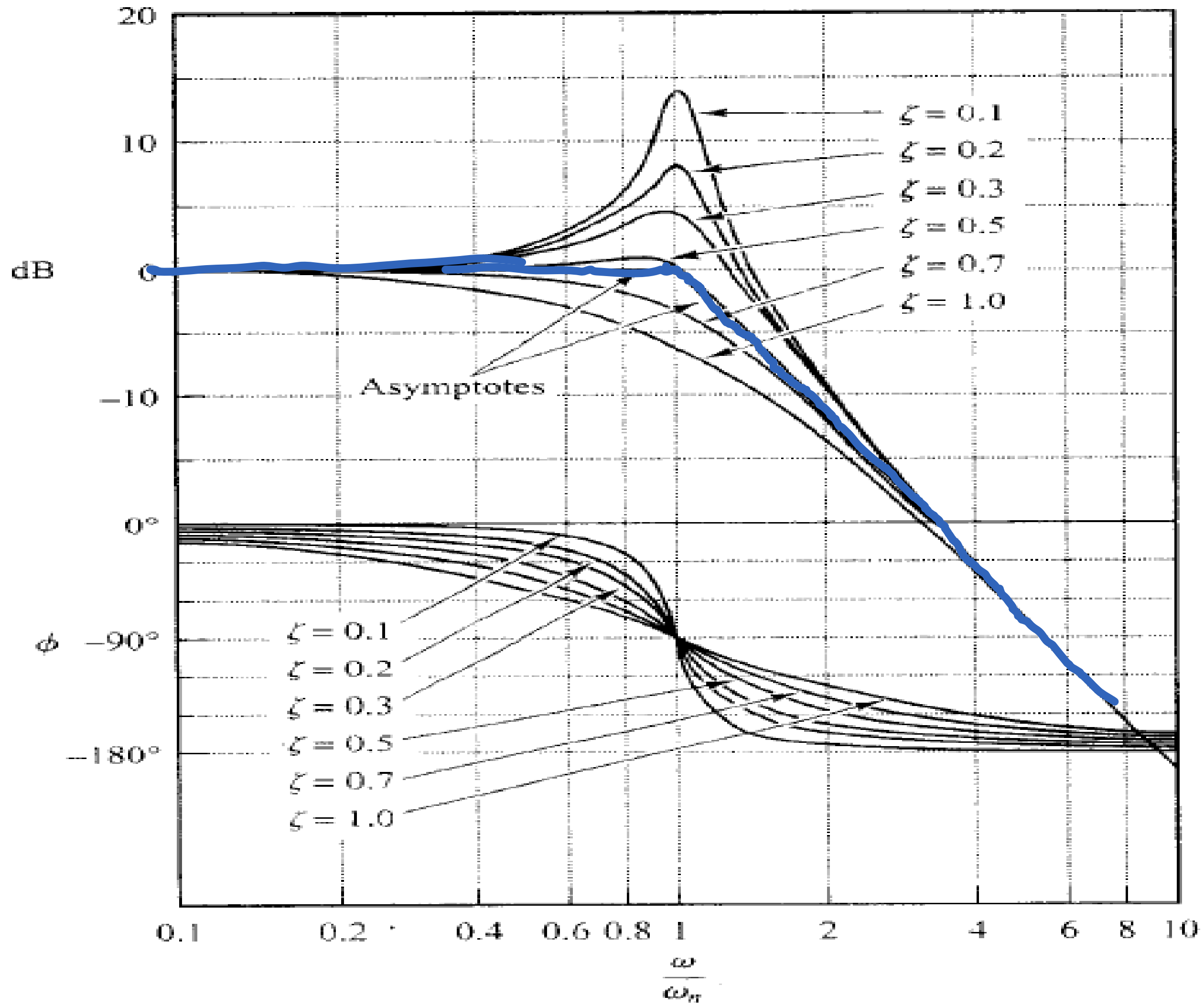
$$M=20\log|G(j\omega)|=-20\log\left|1-\frac{\omega^2}{\omega_n^2}+\frac{2\xi\omega}{\omega_n}j\right|$$

$$=-20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}\Rightarrow\begin{cases}0 & \omega/\omega_n\ll 1\\ -40\log\omega/\omega_n & \omega/\omega_n\gg 1\end{cases}$$

$$\varphi=-tg^{-1}\frac{2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\Rightarrow\begin{cases}0 & \omega/\omega_n\ll 1\\ -90 & \omega/\omega_n=1\\ -180 & \omega/\omega_n\gg 1\end{cases}$$


0
-20
-40
0.1
1
10
ω/ω_n
$\xi=0.7$
0.5
-40


0
1
-90
-180
ω/ω_n


20
10
dB
0
−10
0°
ϕ −90°
−180°
ζ = 0.1
ζ = 0.2
ζ = 0.3
ζ = 0.5
ζ = 0.7
ζ = 1.0
Asymptotes
ζ = 0.1
ζ = 0.2
ζ = 0.3
ζ = 0.5
ζ = 0.7
ζ = 1.0
0.1
0.2
0.4
0.6
0.8
1
2
4
6
8
10
ω/ωn

دانشگاه سمنان

کنترل اتوماتیک

تحلیل پاسخ فرکانسی
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The Resonant Frequency ω_r and the Resonant Peak Value M_r. The magnitude of

$$G(j\omega) = \frac{1}{1 + 2\zeta\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}$$

is

دامنه عامل مرتبه 2

$$|G(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \tag{8–9}$$

If $|G(j\omega)|$ has a peak value at some frequency, this frequency is called the *resonant frequency*. Since the numerator of $|G(j\omega)|$ is constant, a peak value of $|G(j\omega)|$ will occur when

$$g(\omega) = \left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2 \tag{8–10}$$

is a minimum. Since Equation (8–10) can be written

$$g(\omega) = \left[\frac{\omega^2 - \omega_n^2(1 - 2\zeta^2)}{\omega_n^2}\right]^2 + 4\zeta^2(1 - \zeta^2) \tag{8–11}$$

ماکزیمم دامنه در ω_n رخ نمی دهد

the minimum value of $g(\omega)$ occurs at $\omega = \underline{\omega_n\sqrt{1 - 2\zeta^2}}$. Thus the resonant frequency ω_r is

$$\omega_r = \omega_n\sqrt{1 - 2\zeta^2}, \qquad \text{for } 0 \le \zeta \le 0.707 \tag{8–12}$$

As the damping ratio ζ approaches zero, the resonant frequency approaches ω_n. For $0 < \zeta \le 0.707$, the resonant frequency ω_r is less than the damped natural frequency $\omega_d = \omega_n\sqrt{1 - \zeta^2}$, which is exhibited in the transient response. From Equation (8–12), it can be seen that for $\zeta > 0.707$, there is no resonant peak. The magnitude $|G(j\omega)|$ decreases monotonically with increasing frequency ω. (The magnitude is less than 0 dB for all values of $\omega > 0$. Recall that, for $0.7 < \zeta < 1$, the step response is oscillatory, but the oscillations are well damped and are hardly perceptible.)

The magnitude of the resonant peak, M_r, can be found by substituting Equation (8–12) into Equation (8–9). For $0 \leq \zeta \leq 0.707$,

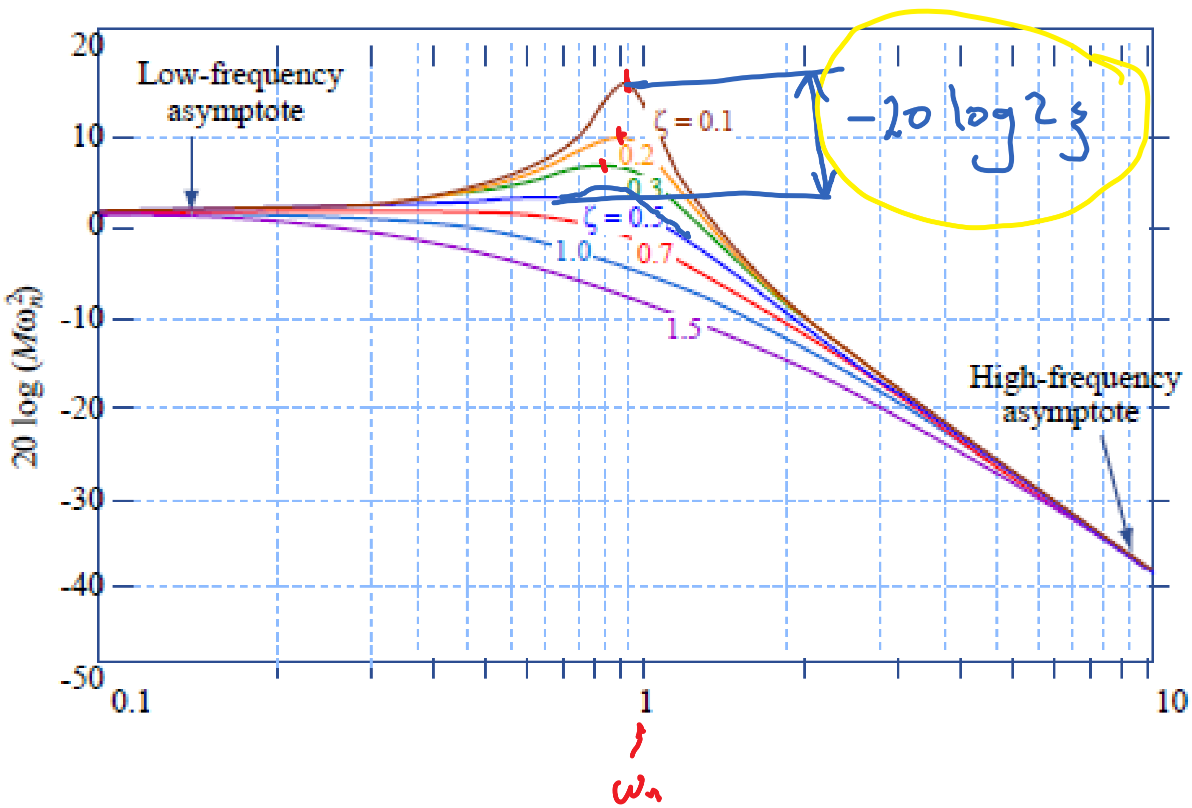
$$M_r = |G(j\omega)|_{\max} = |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad (8\text{–}13)$$

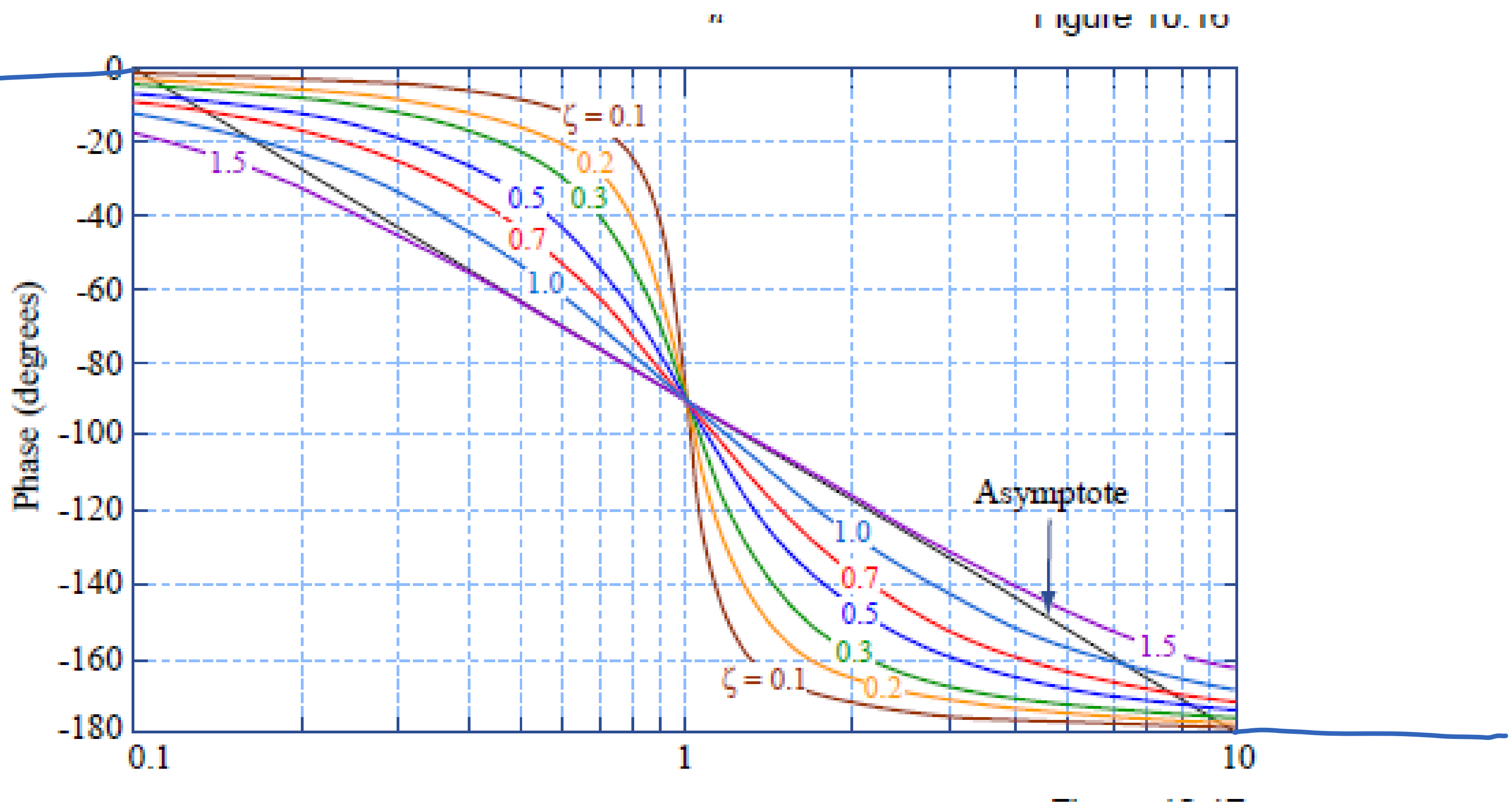
For $\zeta > 0.707$,

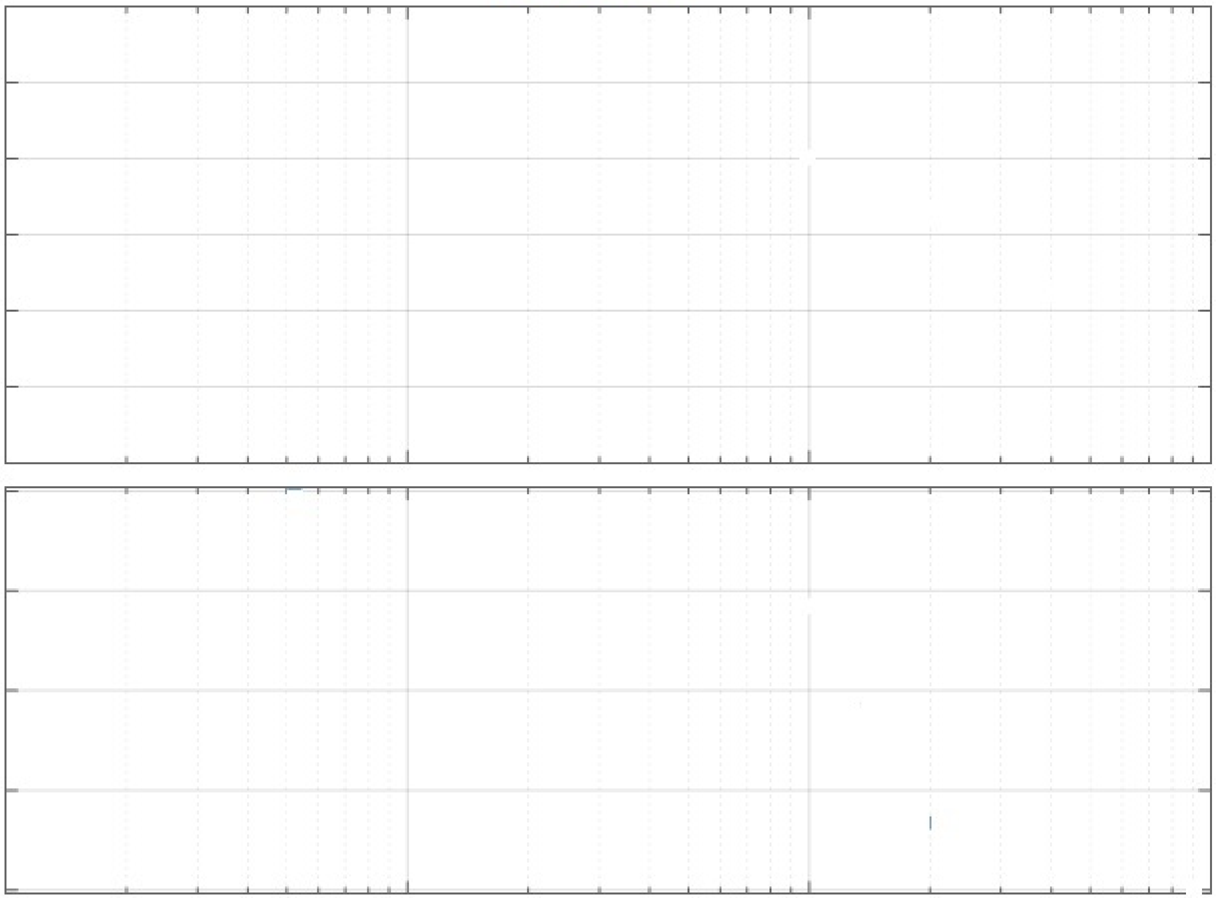
$$M_r = 1 \qquad (8\text{–}14)$$

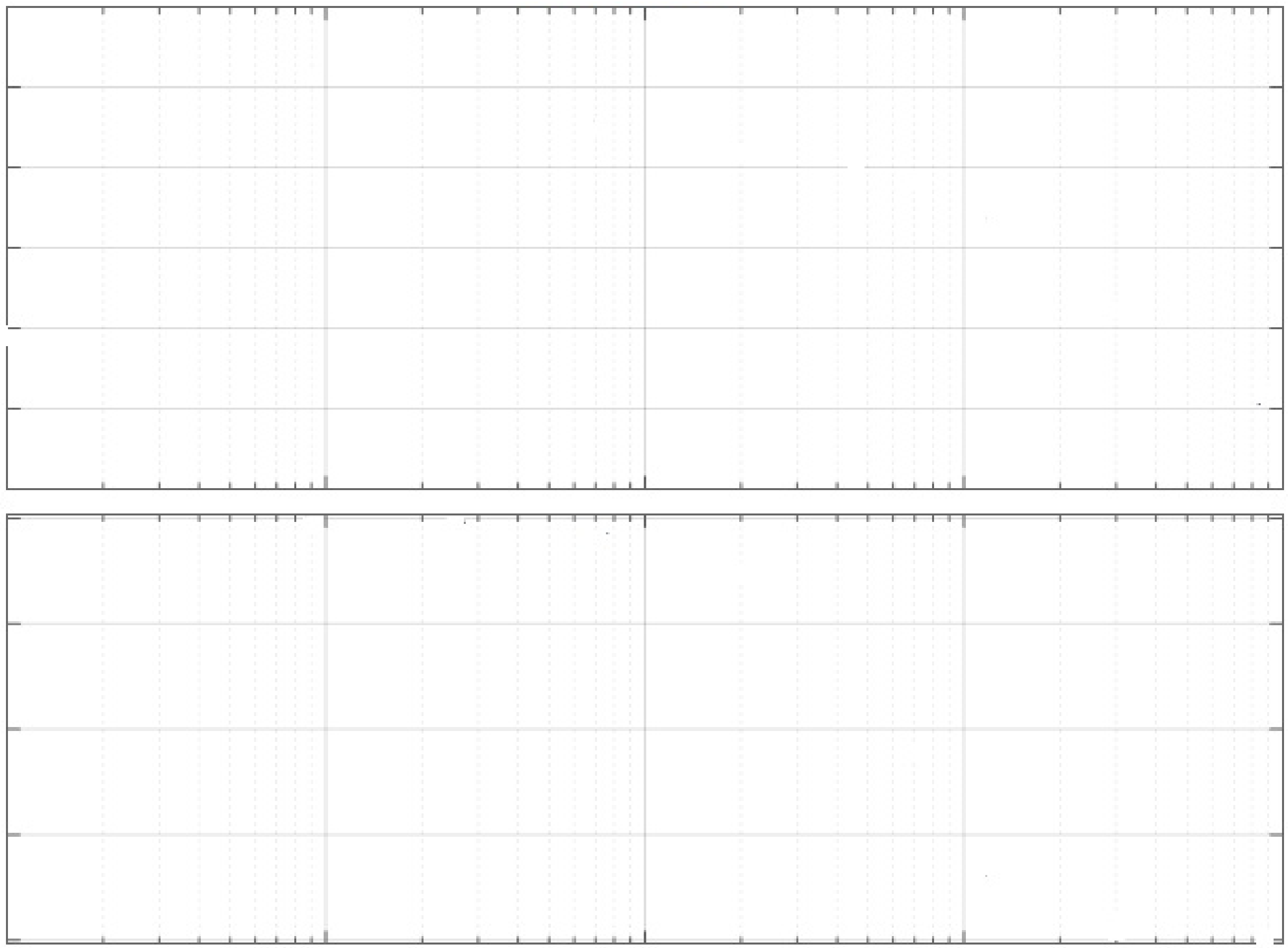
As ζ approaches zero, M_r approaches infinity. This means that if the undamped system is excited at its natural frequency, the magnitude of $G(j\omega)$ becomes infinity. The relationship between M_r and ζ is shown in Figure 8–10.

$$20 \log M_r = 20 \log \frac{1}{2\zeta\sqrt{1-\zeta^2}} = -20 \log 2\zeta\sqrt{1-\zeta^2} \simeq \boxed{-20 \log 2\zeta}$$


Low-frequency
asymptote
High-frequency
asymptote
$\zeta = 0.1$
0.2
0.3
$\zeta = 0.5$
0.7
1.0
1.5
$20 \log (M\omega_n^2)$
20
10
0
-10
-20
-30
-40
-50
0.1
1
10
$-20 \log 2\zeta$
ω_n


Phase (degrees)
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
0.1
1
10
ζ = 0.1
0.2
0.3
0.5
0.7
1.0
1.5
Asymptote

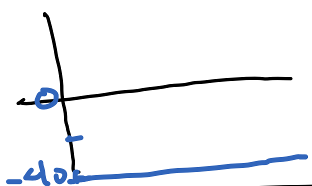
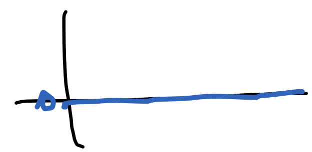
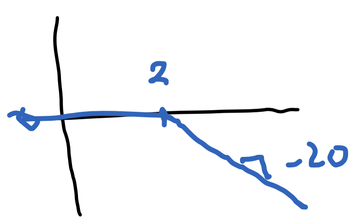
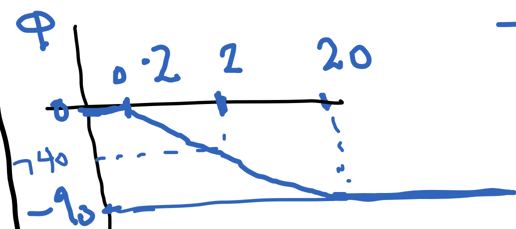
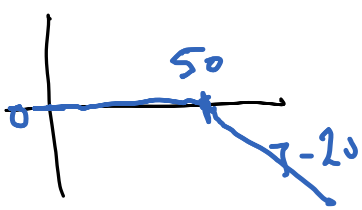
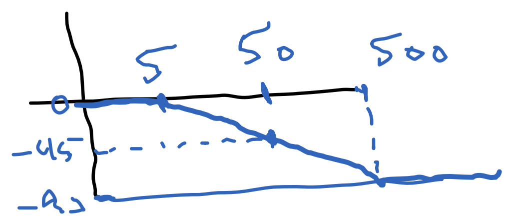




$Ts+1$

$$G(s)=\frac{1}{(s+2)(s+50)}=1 \qquad G(s)=\frac{1}{2\times 50\left(\frac{s}{2}+1\right)\left(\frac{s}{50}+1\right)}=\frac{0.01}{\left(\frac{s}{2}+1\right)\left(\frac{s}{50}+1\right)}$$

حل مرحله 1

	دامنه M	فاز φ
$G_1=0.01$		
$G_2=\frac{1}{s/2+1}$		
$G_3=\frac{1}{s/50+1}$		

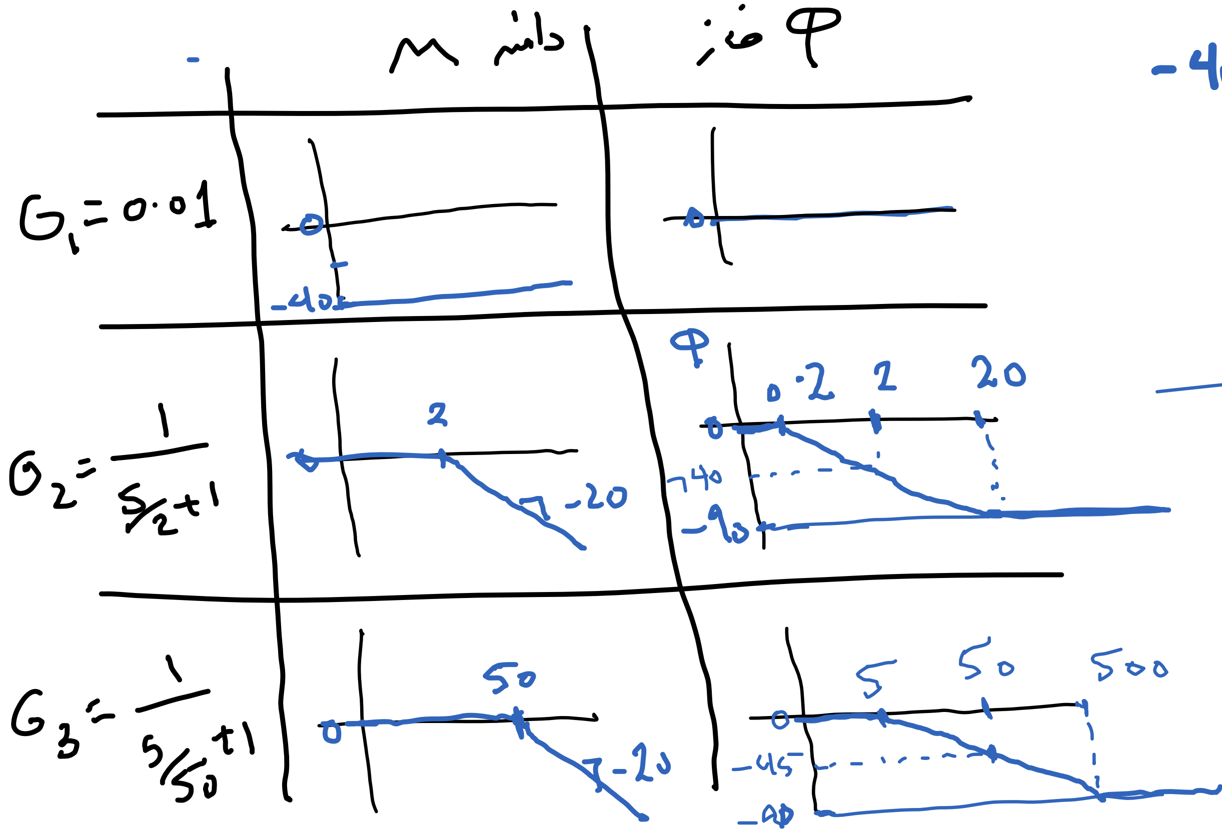
$$M_1=20\log|G_1|=20\log 0.01=-40$$

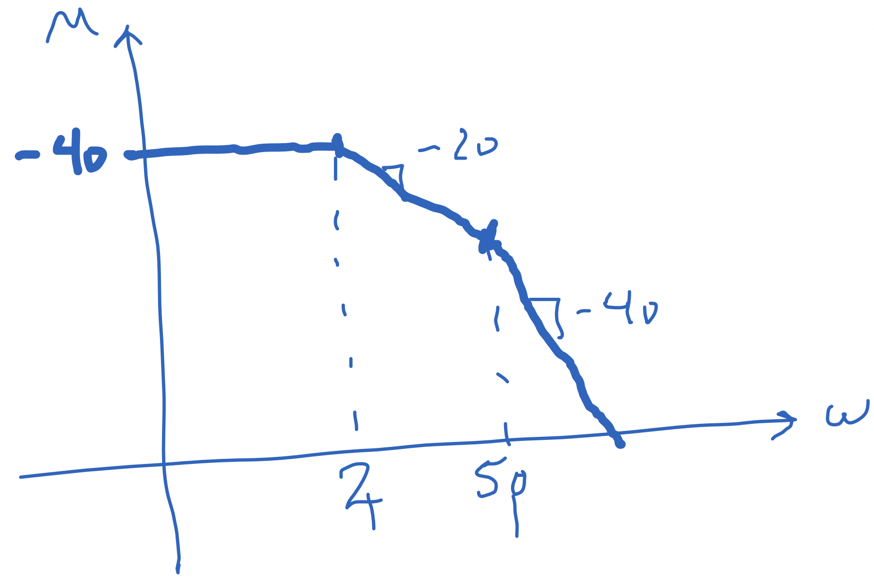
$$G_2,\ T=\frac{1}{2}\longrightarrow \text{فرکانس گوشه}\ \frac{1}{T}=2$$

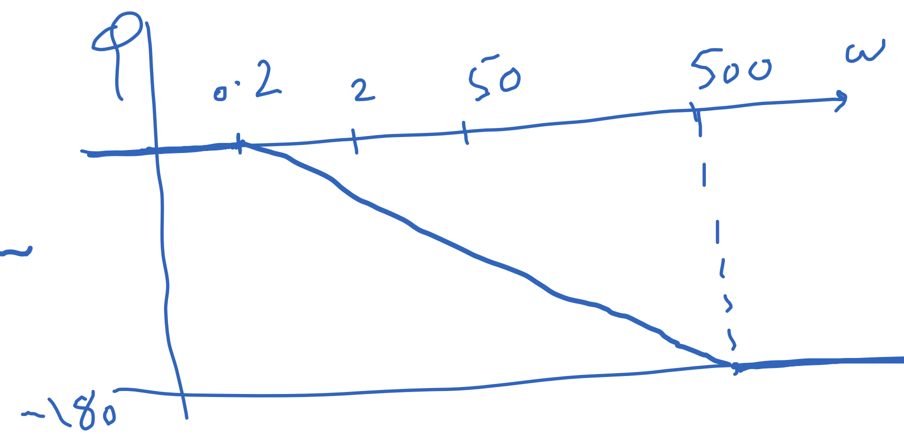
فرکانس های گوشه ای

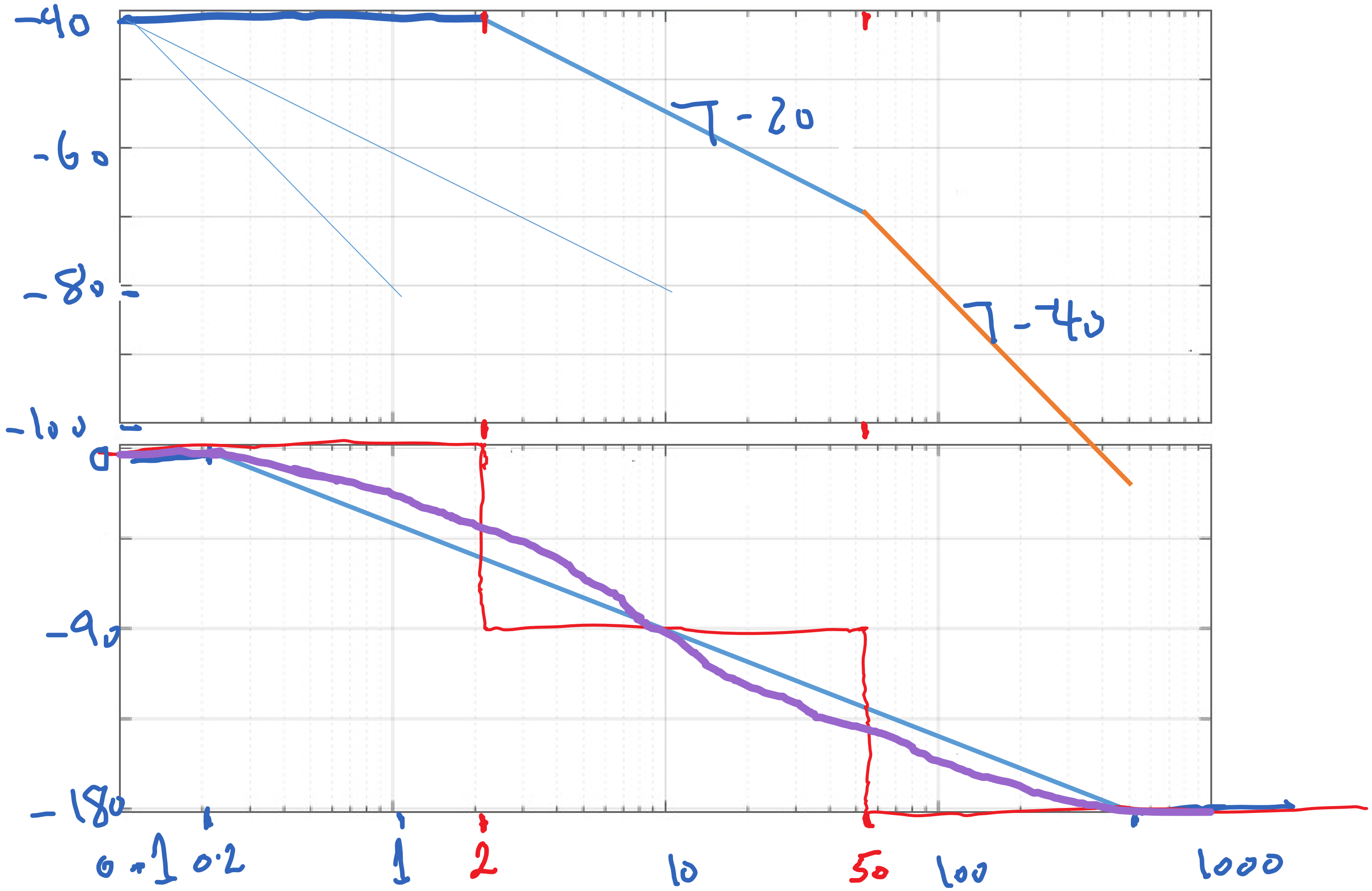
0.1 0.2 2 50 500 1000

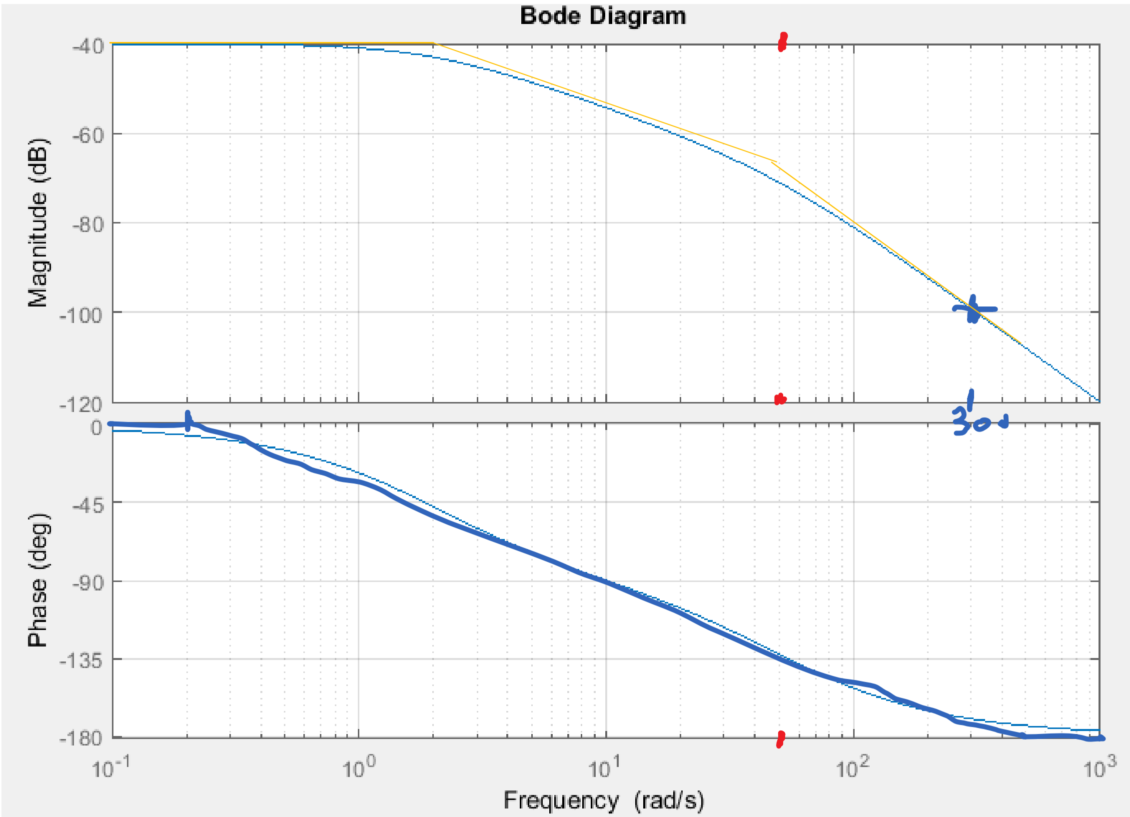
0.1 1 10 100 1000


دامنه M
فاز φ
$G_1 = 0.01$
0
-40
0
$G_2 = \frac{1}{s/2+1}$
2
-20
φ
0.2
2
20
0
-45
-90
$G_3 = \frac{1}{s/50+1}$
50
0
-20
5
50
500
0
-45
-90


M
-40
-20
-40
ω
2
50


φ
0.2
2
50
500
ω
-180

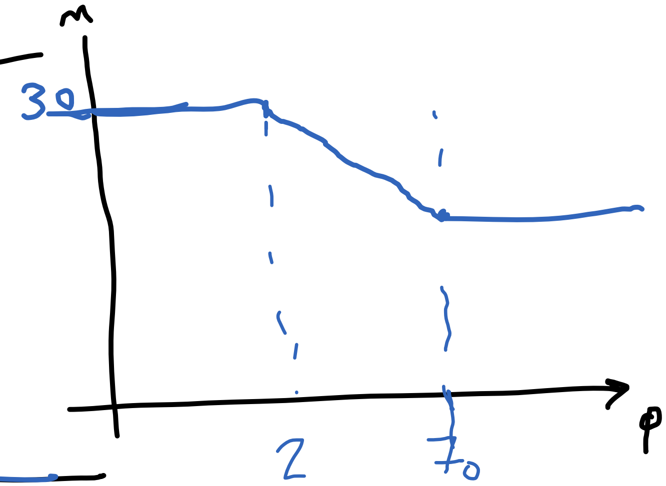

-40
-60
-80
-100
-20
-40
0
-90
-180
0.1
0.2
1
2
10
50
100
1000



$$\frac{1}{(s+2)(s+50)}$$

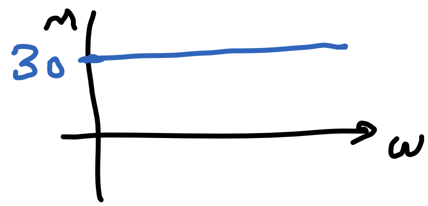
$$G(s) = \frac{1}{s^2 + 52s + 100}$$

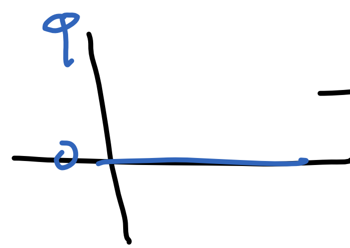
$$G(s) = \frac{(s+70)}{(s+2)} = \frac{70\left(\frac{s}{70}+1\right)}{2\left(\frac{s}{2}+1\right)} = \frac{35\left(\frac{s}{70}+1\right)}{\left(\frac{s}{2}+1\right)}$$

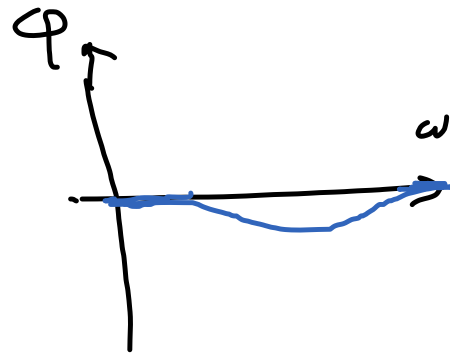


0.1 0.2 2 70 700 1000

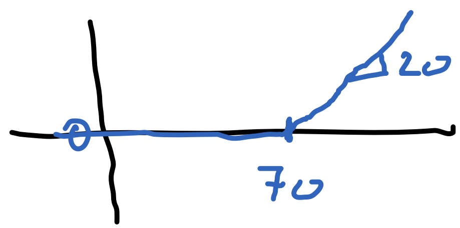
$G_1(s) = 35 \rightarrow$

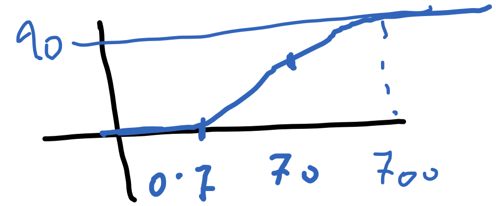




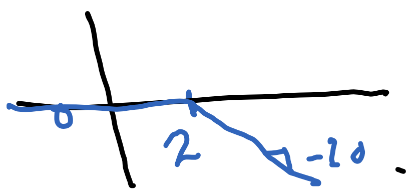


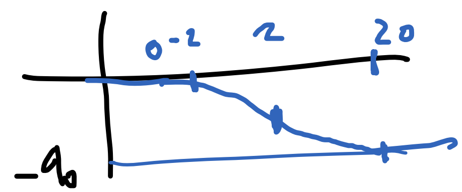
$G_1(s) = \frac{s}{70}+1$

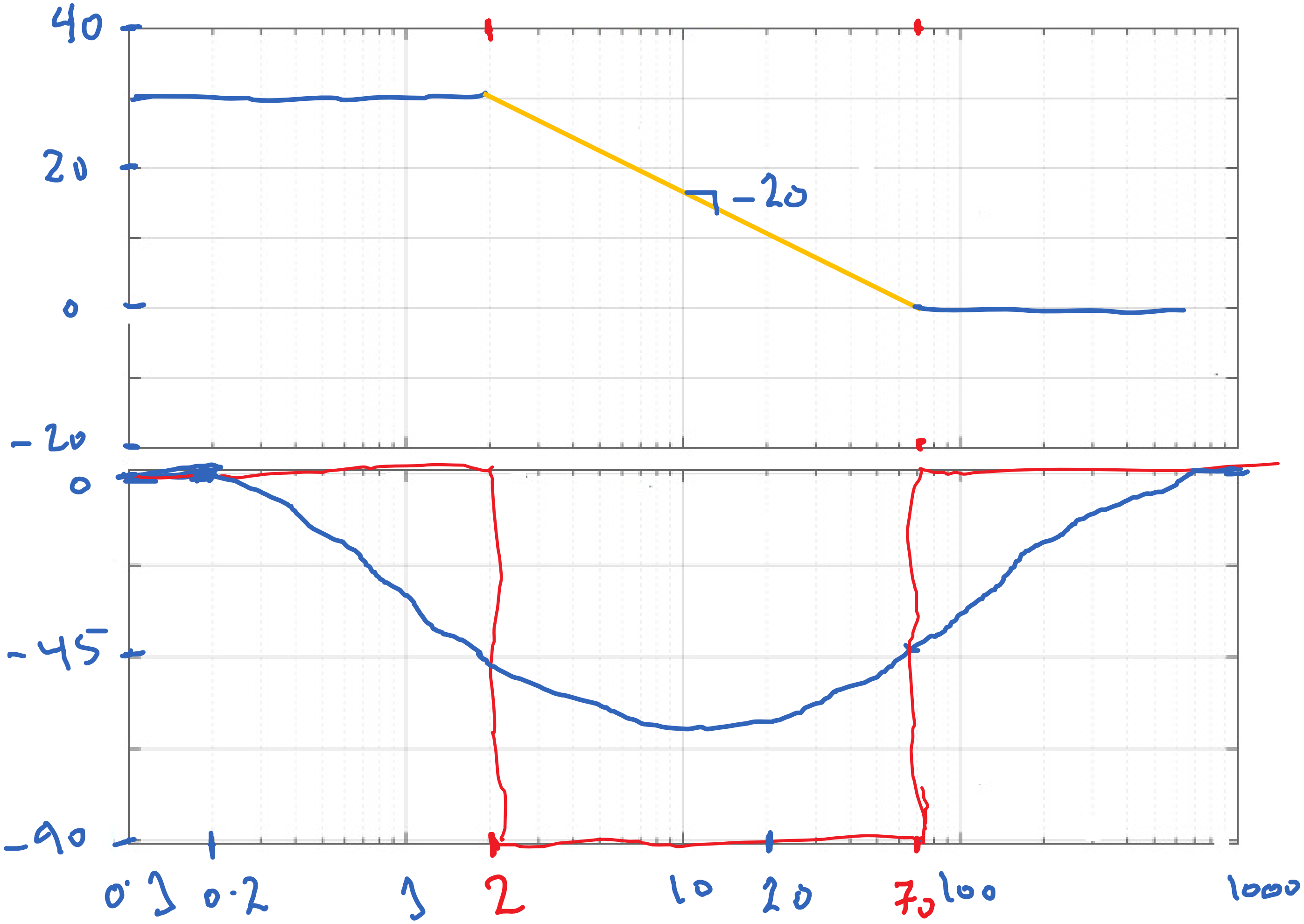


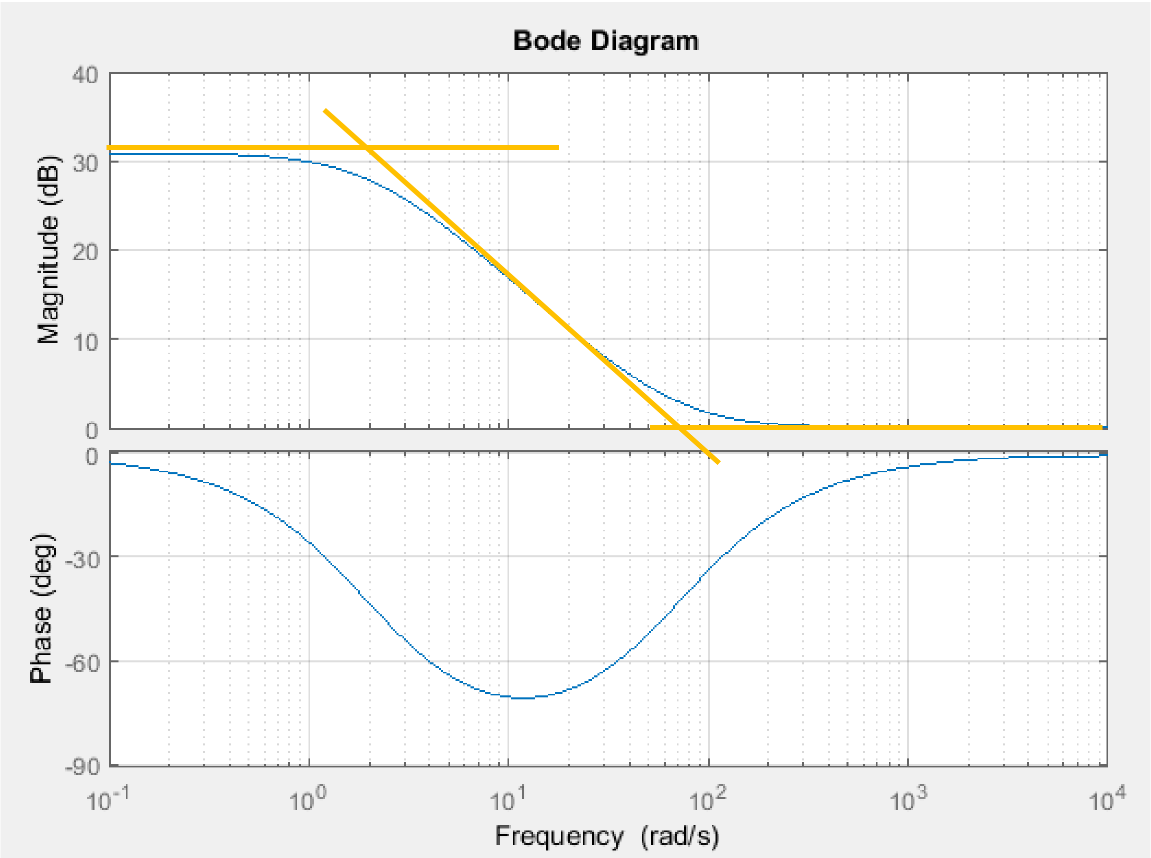


$G_2(s) = \frac{1}{\frac{s}{2}+1}$





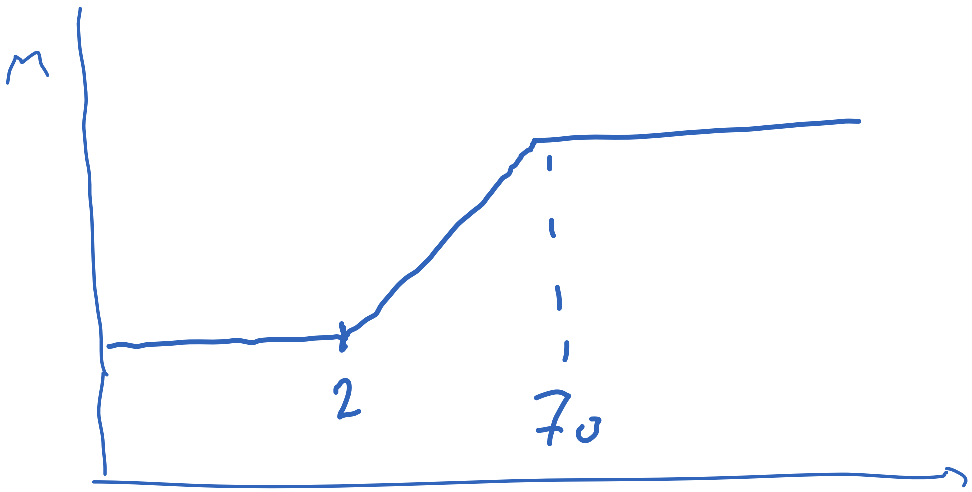

40
20
0
-20
-20
0
-45
-90
0.1
0.2
1
2
10
20
70
100
1000

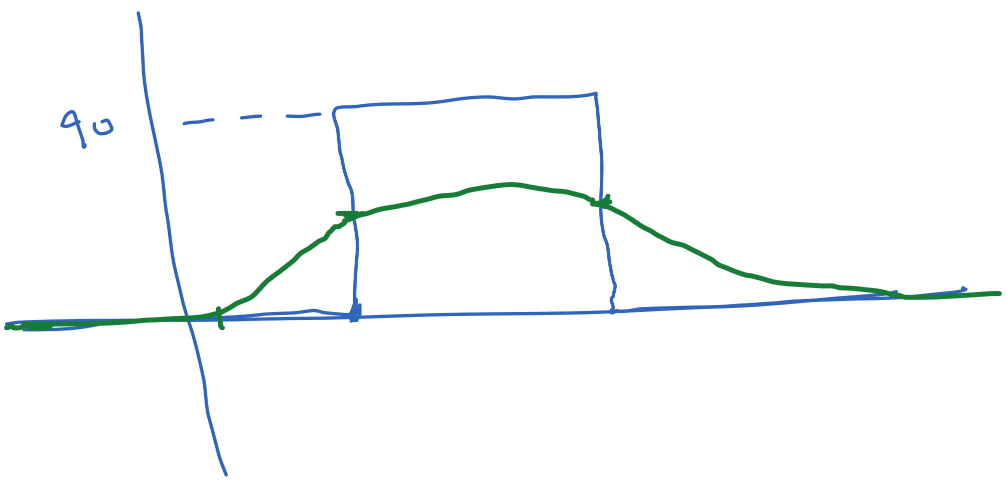

Bode Diagram
Magnitude (dB)
40
30
20
10
0
Phase (deg)
0
-30
-60
-90
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/s)

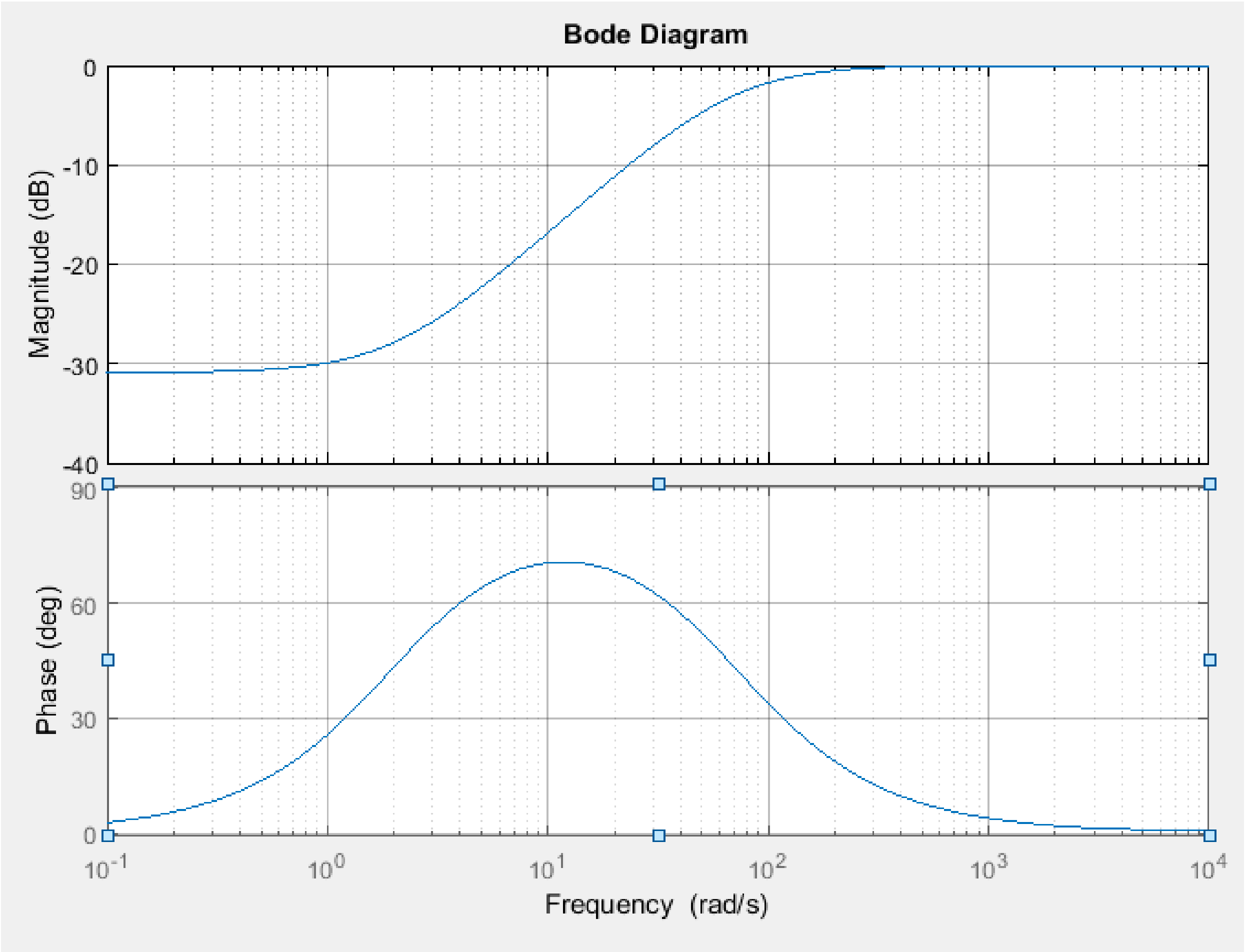
$$\frac{(s+70)}{(s+2)}$$

(s+2)

(s+70)



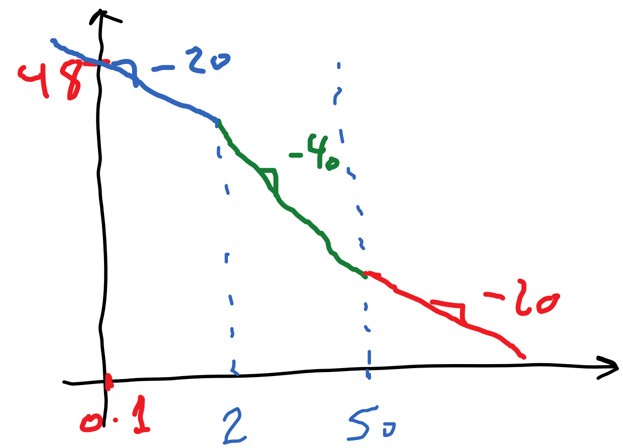



Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
90
60
30
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/s)

(s+2)

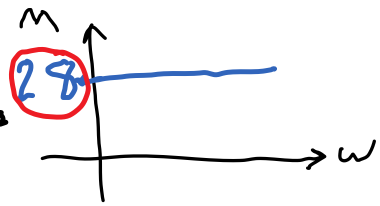
(s+70)

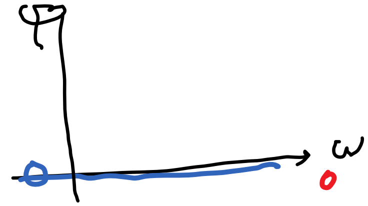
$$G(s) = \frac{(s+50)}{s\,(s+2)} = \frac{50\left(\frac{s}{50}+1\right)}{2s\left(\frac{s}{2}+1\right)} = \frac{25\left(\frac{s}{50}+1\right)}{s\left(\frac{s}{2}+1\right)}$$



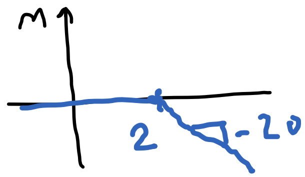
$G_1(s) = 25 \rightarrow$

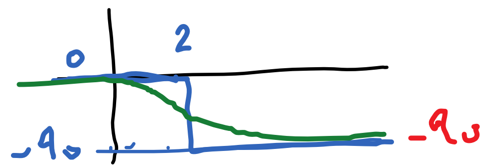
$20 \log 25 = 28$

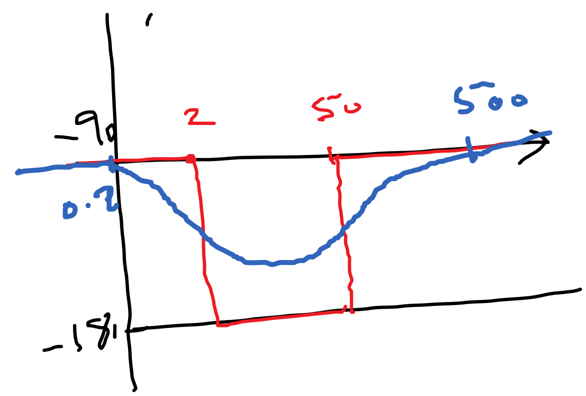




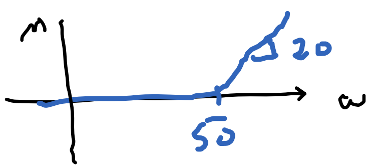
$G_2(s) = \frac{1}{s/2 + 1}$

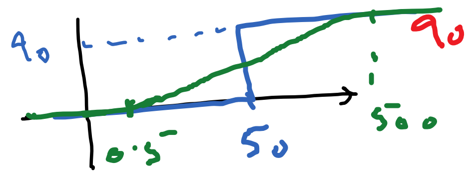




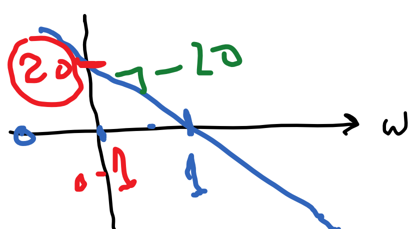


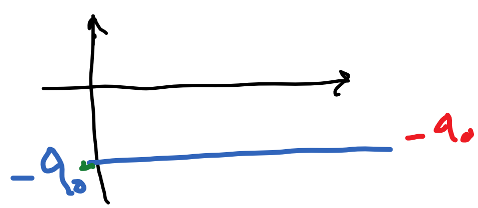
$G_3(s) = \frac{s}{50} + 1$



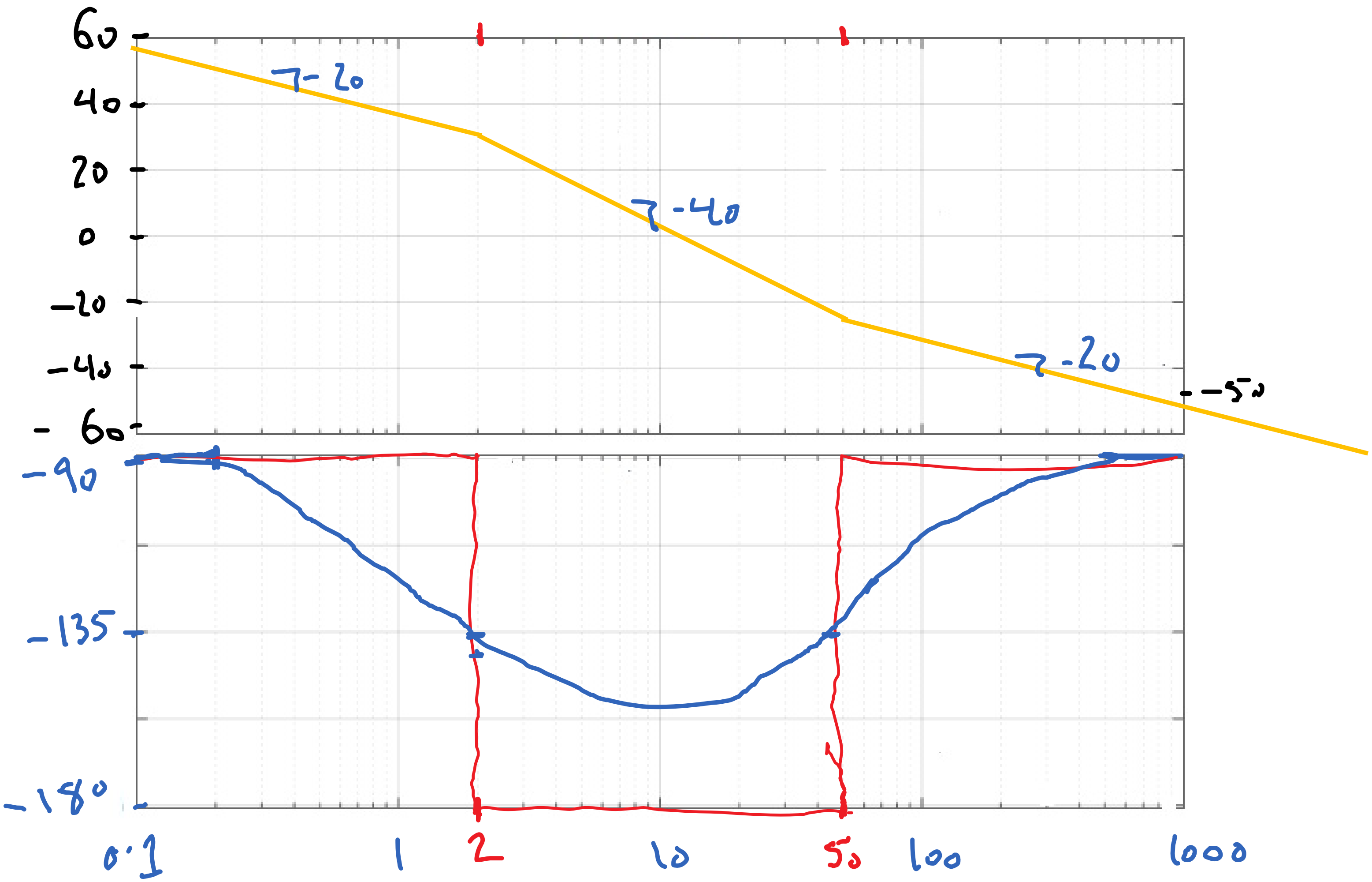


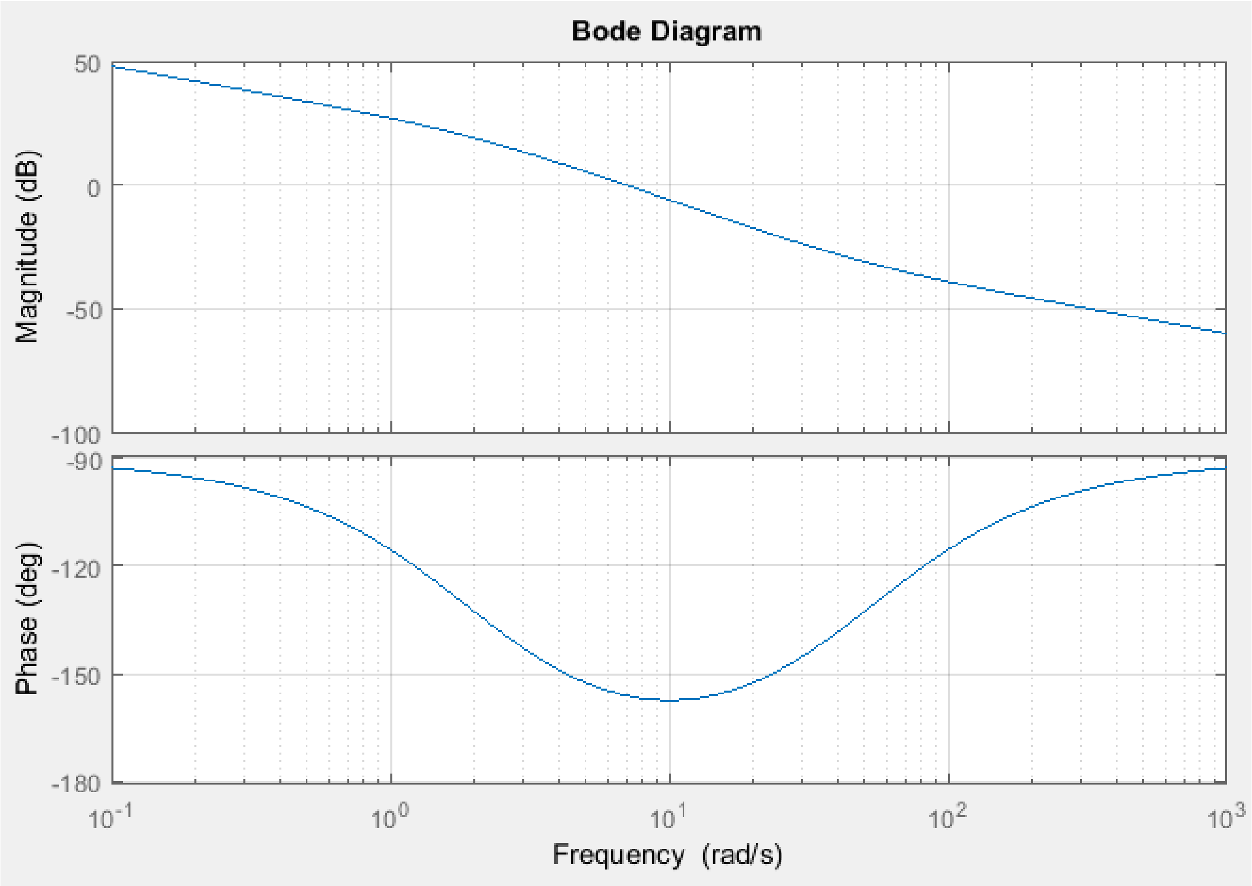
$G_4 = \frac{1}{s}$





$-90 \rightarrow -90$

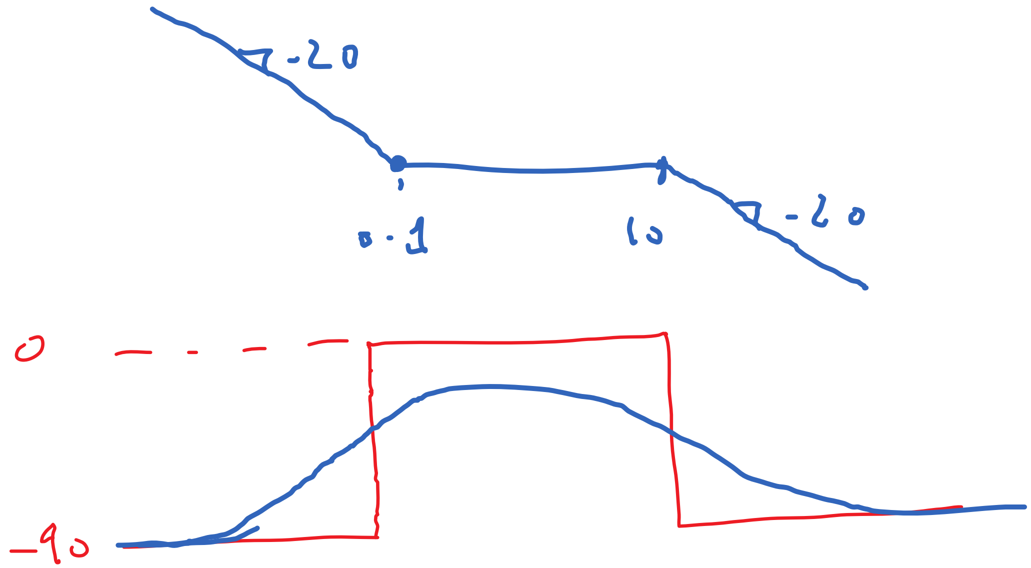

60
40
20
0
-20
-40
-60
-20
-40
-20
-50
-90
-135
-180
0.1
1
2
10
50
100
1000


Bode Diagram
Magnitude (dB)
50
0
-50
-100
Phase (deg)
-90
-120
-150
-180
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/s)

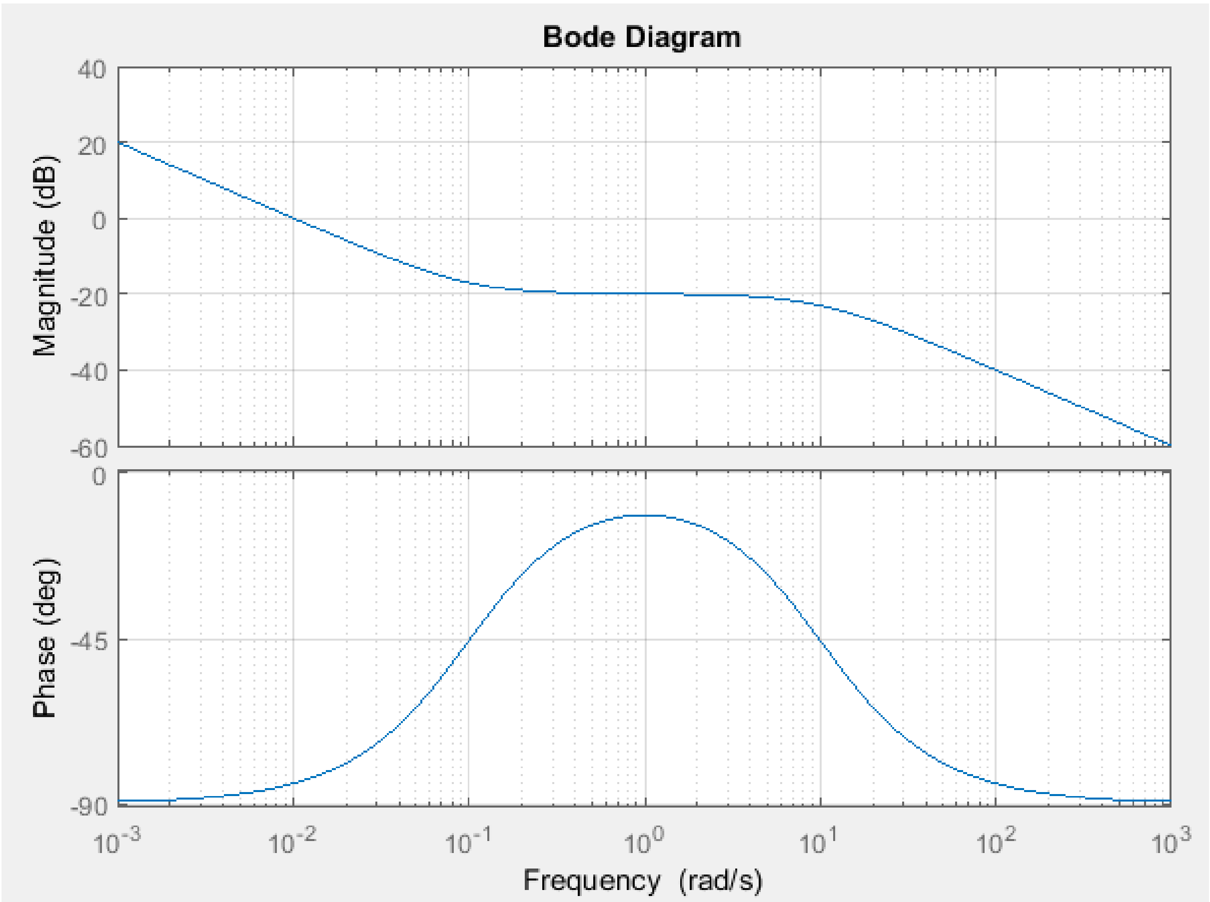
$$\frac{(s+50)}{s\,(s+2)}$$

$$\frac{(s+0.1)}{s\,(s+10)}$$

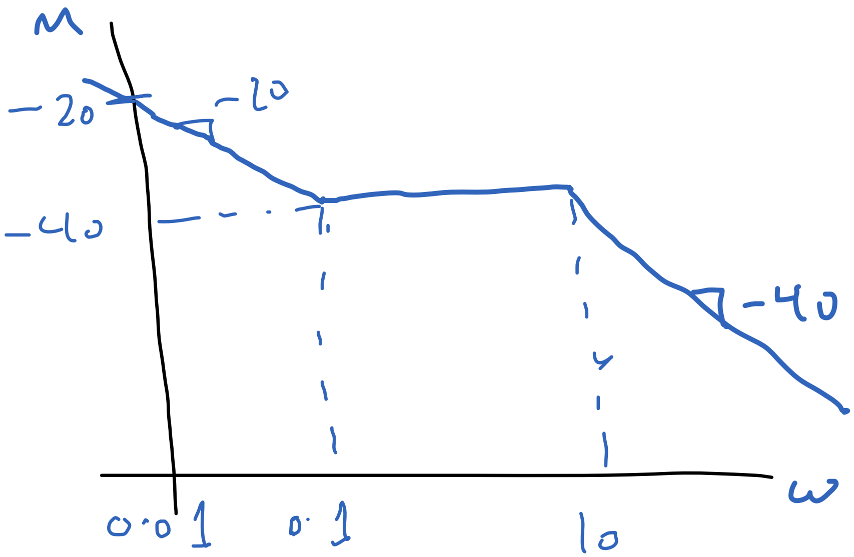
0.1 , 10

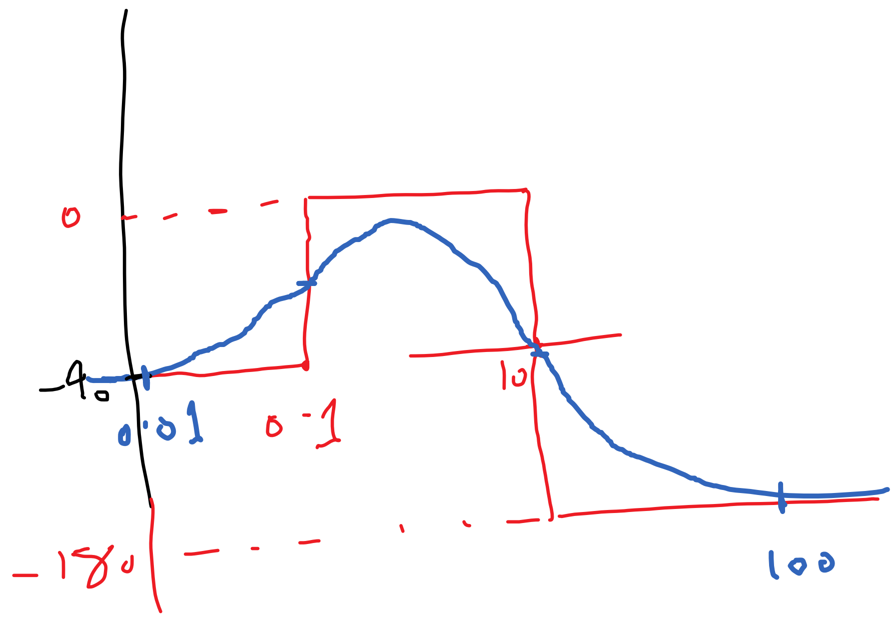


$$\frac{(s+0.1)}{s\,(s+10)}$$

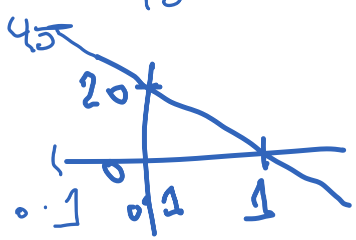


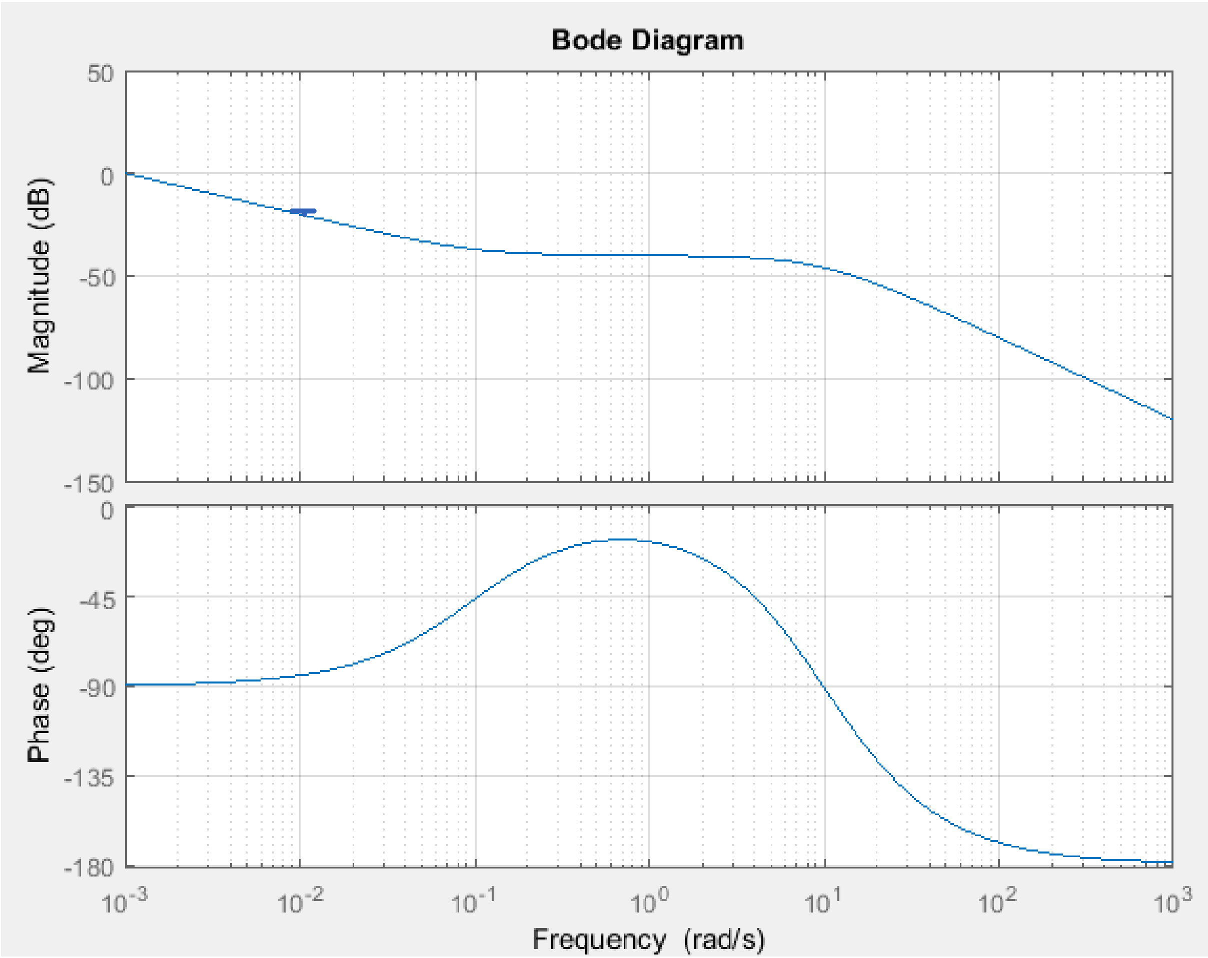
$$\frac{(s+0.1)}{s\,(s+10)^2} = \frac{0.1\left(\frac{s}{0.1}+1\right)}{10\times 10\, s\left(\frac{s}{10}+1\right)^2} = 10^{-3}\,\frac{\left(\frac{s}{0.1}+1\right)}{s\left(\frac{s}{10}+1\right)^2}$$



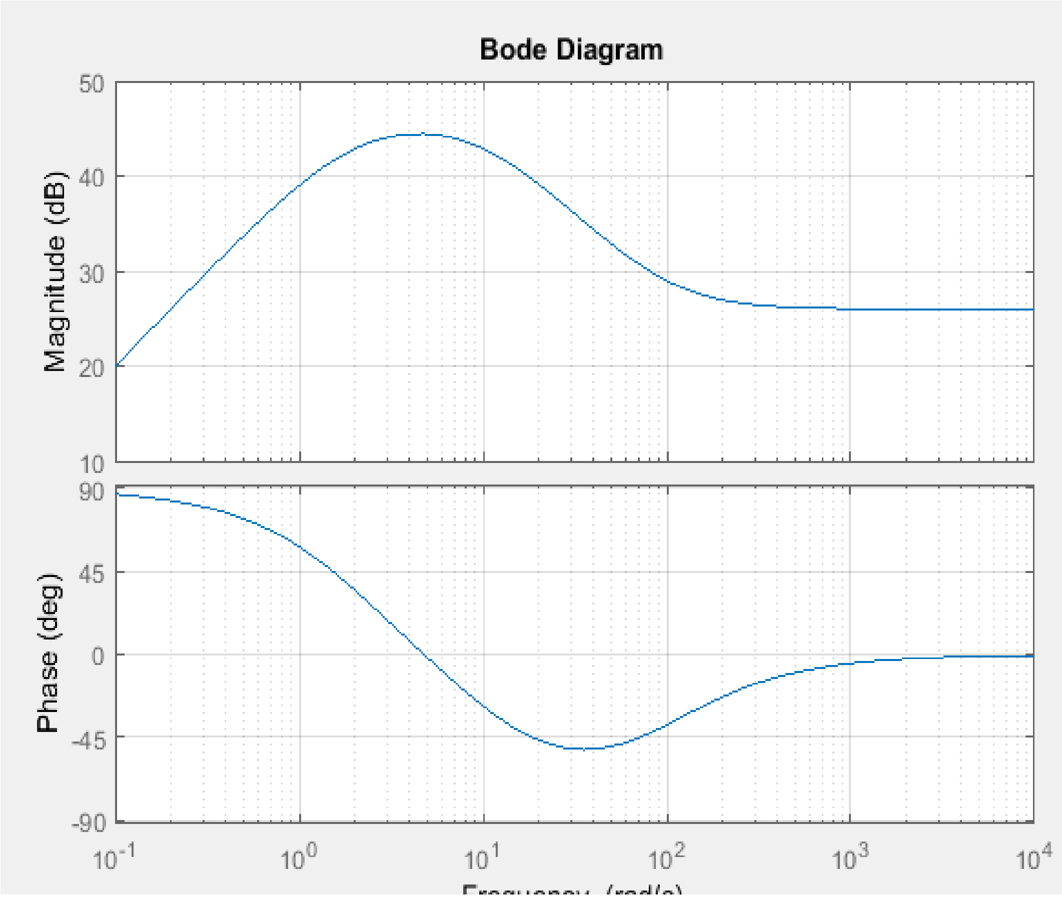


$$20\log_{10} 10^{-3} = -60$$




Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
Phase (deg)
0
-45
-90
-135
-180
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

 (s+0.1)
 ----
s (s+10)^2



$$\frac{20\, s\, (s+100)}{(s+2)\,(s+10)}$$

$$G(s) = \frac{(s+10)}{(s+1)(s^2+40s+10^4)} = \frac{10}{10^4}\frac{\left(\frac{s}{10}+1\right)}{(s+1)\left(\frac{s^2}{10^4}+\frac{40s}{10^4}+1\right)}$$

$$\frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1}$$

The system has one zero at $s=-10$, one real pole at $s=-1$, and one pole pair whose natural frequency $\omega_n = 100$ (rad/s) and damping ratio $\zeta = 40/2/\omega_n = 0.2$. Hence, the system has three break frequencies: $\underline{1}$, $\underline{10}$, $\underline{100}$ (rad/s).

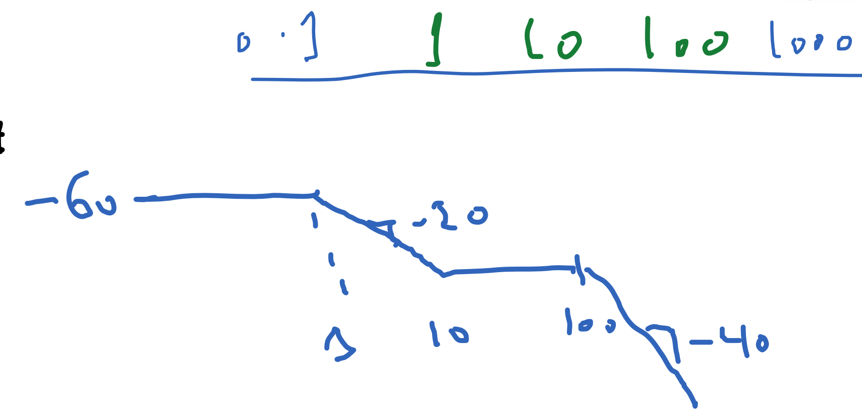
$\omega_n = 100$

$\frac{2\zeta}{\omega_n} = \frac{40}{10^4} \rightarrow \zeta = 0.2$

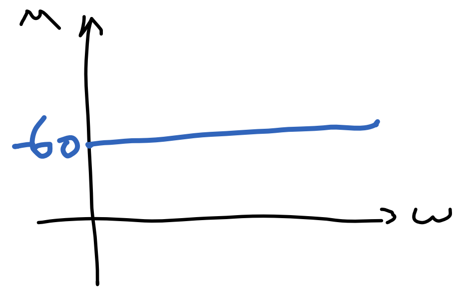
The magnitude at the low frequency asymptote is $20\log_{10} 10^{-3} = -60$ (dB).

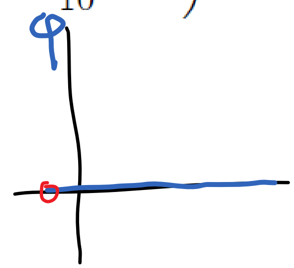
The correction $-20\log 2\zeta = 7.96$ (dB).

$$G(s) = \frac{(s+10)}{(s+1)(s^2+40s+10^4)} = \frac{10}{10^4}\,\frac{\left(\frac{s}{10}+1\right)}{(s+1)\left(\frac{s^2}{10^4}+\frac{40s}{10^4}+1\right)} \rightarrow G_4$$



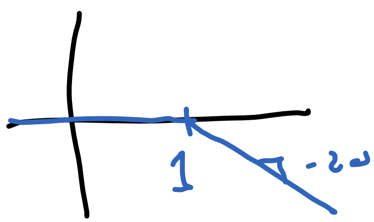
$G_1 = 10^{-3} \rightarrow$

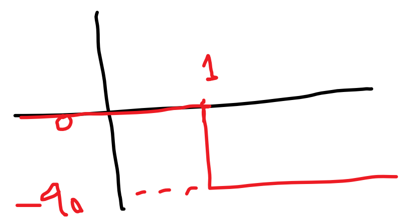


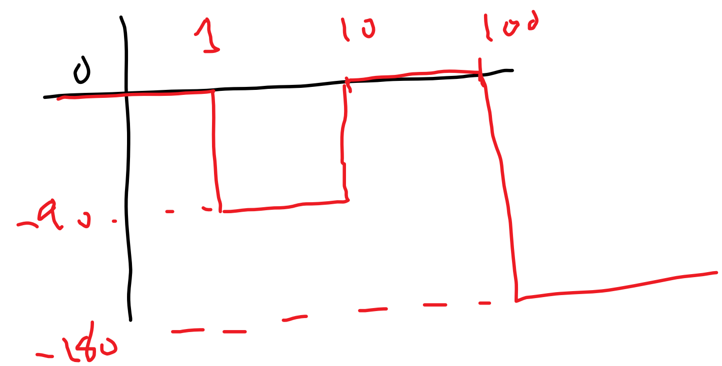


$20\log 10^{-3} = -6$

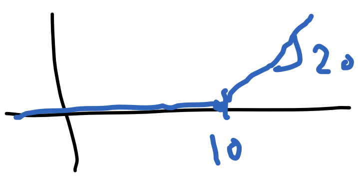
$G_2 = \frac{1}{s+1}$

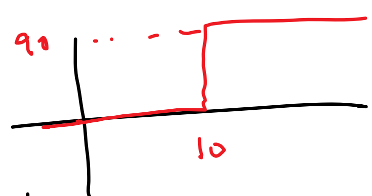




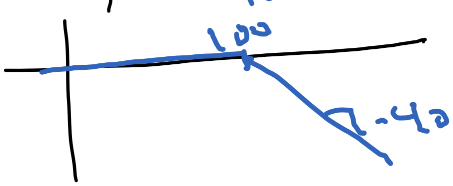


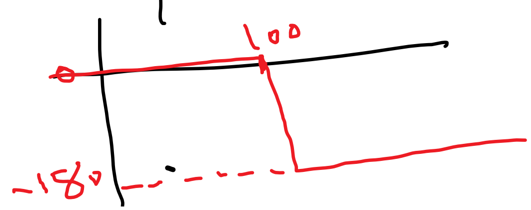
$G_3 = \frac{s}{10}+1$

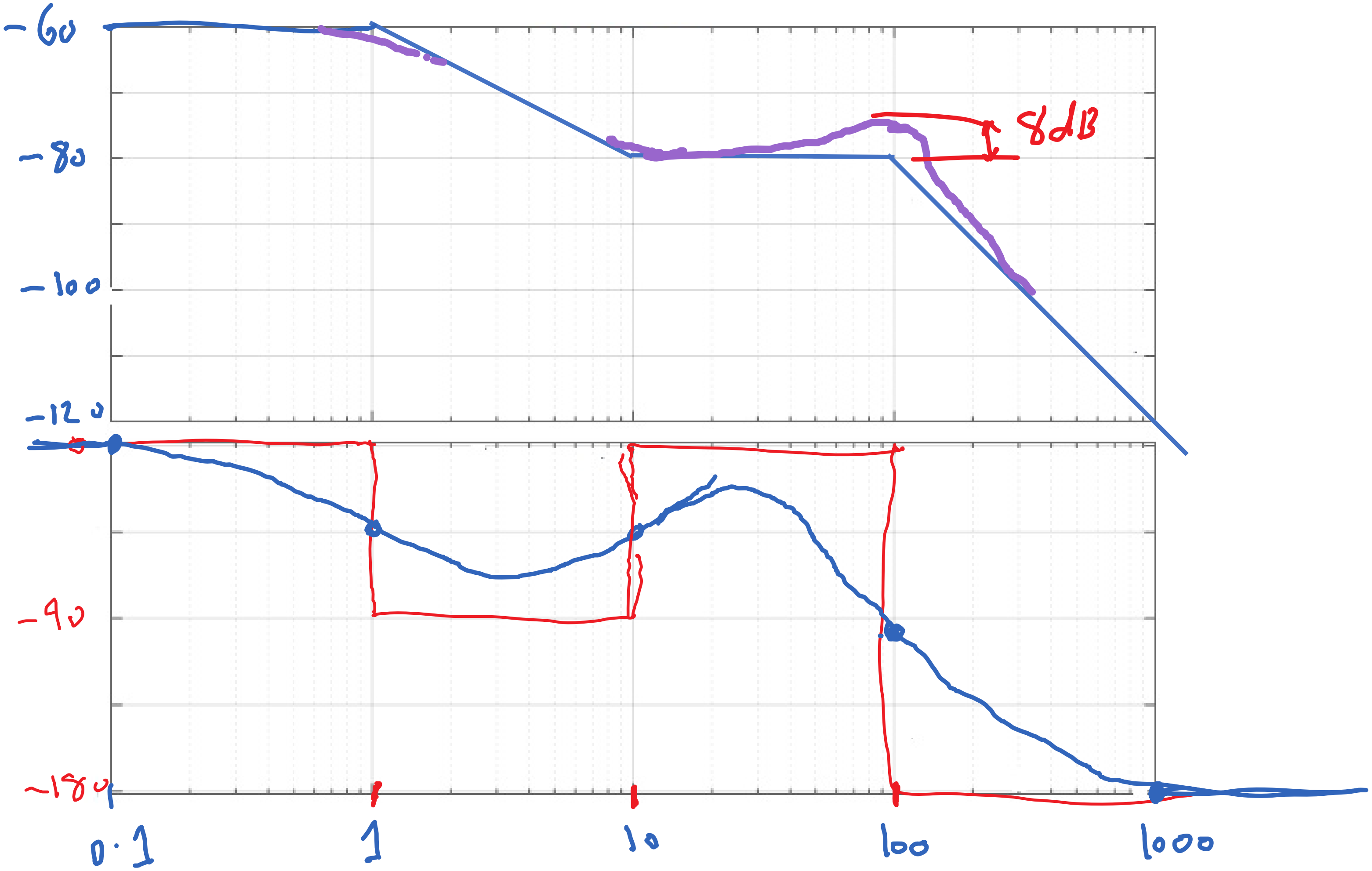


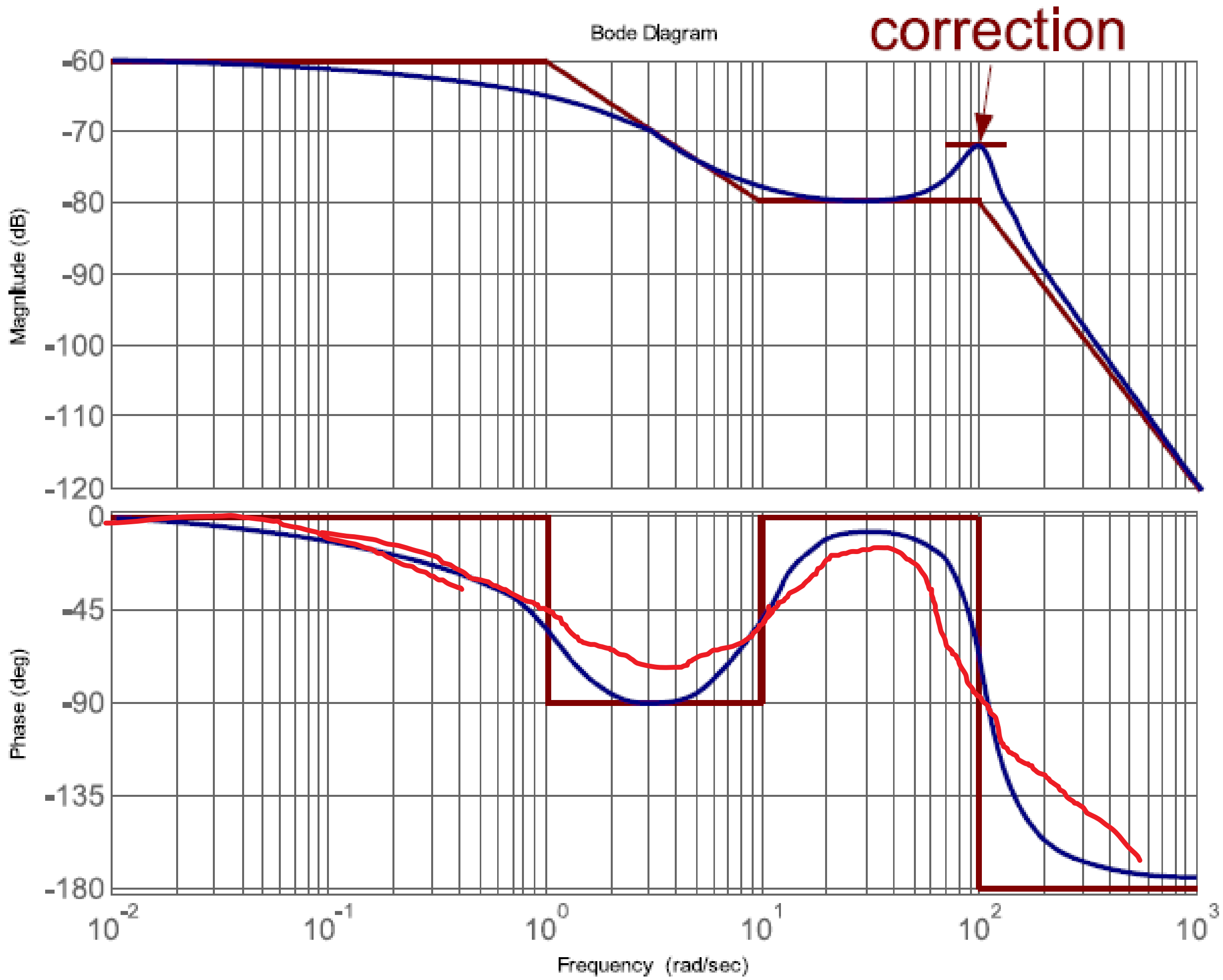


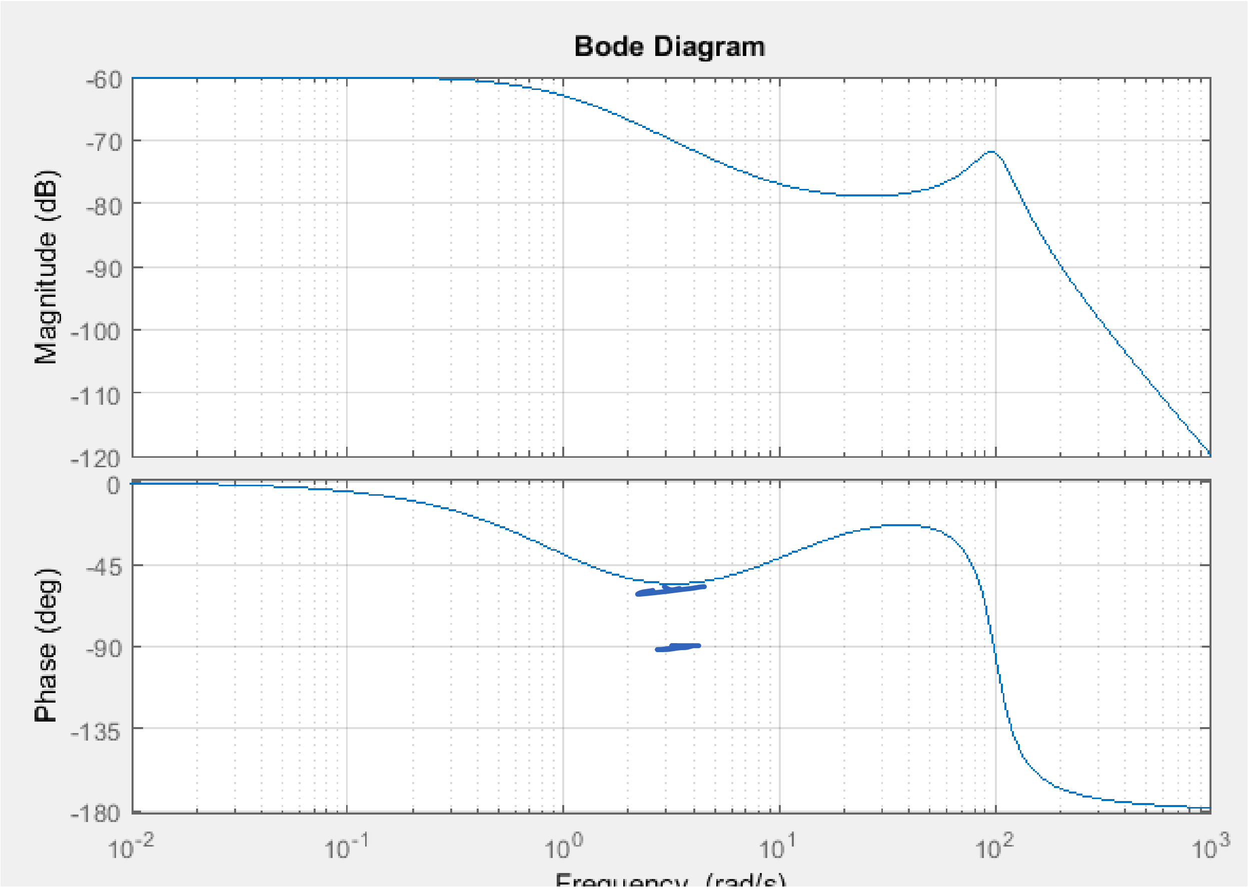
$G_4 =$

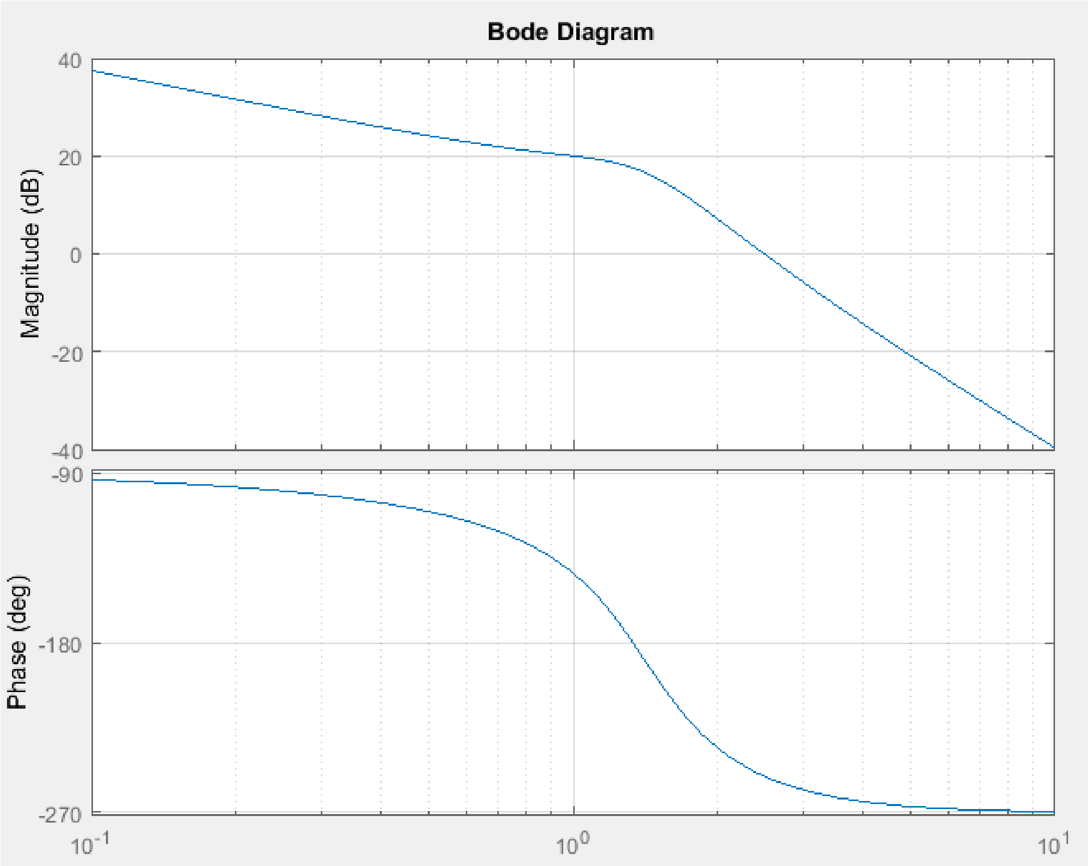





-60
-80
-100
-120
8dB
-90
-180
0.1
1
10
100
1000


Bode Diagram
correction
Magnitude (dB)
-60
-70
-80
-90
-100
-110
-120
Phase (deg)
0
-45
-90
-135
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)


Bode Diagram
Magnitude (dB)
-60
-70
-80
-90
-100
-110
-120
Phase (deg)
0
-45
-90
-135
-180
10^-2
10^-1
10^0
10^1
10^2
10^3
Frequency (rad/s)



$$\frac{10\,(s+3)}{s\,(s+2)\,(s^2+s+2)}$$

کنترل اتوماتیک

تحلیل پاسخ فرکانسی
دیاگرام بود، بخش سوم

دکتر امین نیکوبین

دانشگاه سمنان، دانشکده مهندسی مکانیک

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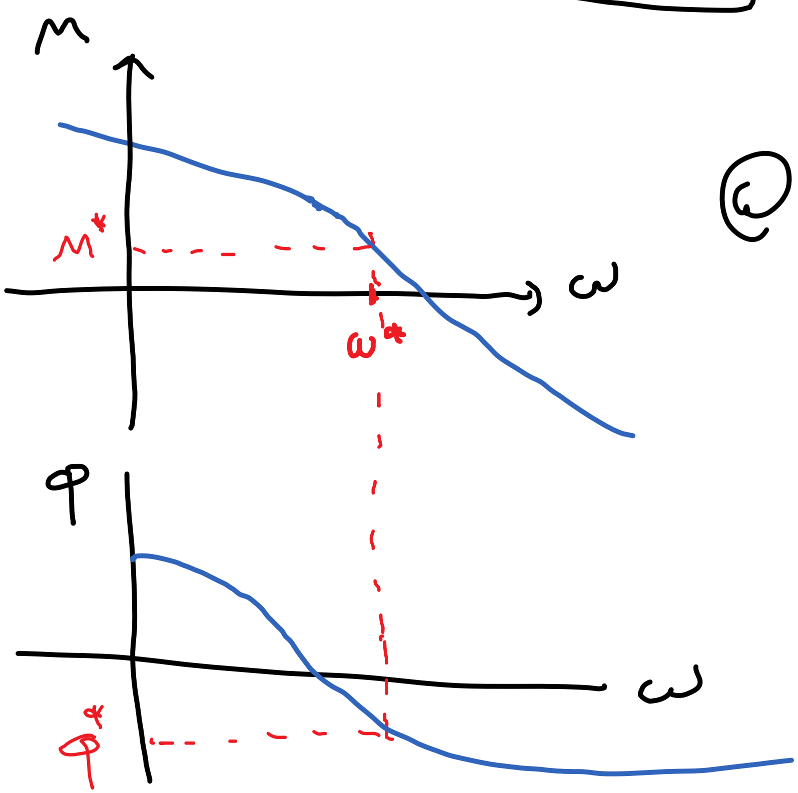
تفسیر منحنی های دامنه و فاز

$$\sin\omega t \longrightarrow \boxed{G(s)} \longrightarrow \quad y_{ss} = \underline{|G(j\omega)|}\,\sin(\omega t + \varphi)$$

$$\varphi = \underline{\angle G(j\omega)}$$



$$@\ \omega^* \Rightarrow M^*,\quad \varphi^*$$

$$M^* = 20\log|G(j\omega)| \Rightarrow \log|G(j\omega)| = \frac{M}{20}$$

$$\Rightarrow |G(j\omega)| = 10^{M^*/20}$$

$$@\ \omega^* \Rightarrow y_{ss} = 10^{M^*/20}\sin(\omega^* t + \varphi^*)$$

$$@\omega^* \Rightarrow y_{ss} = 10^{M^*/20} \sin(\omega^* t + \varphi^*)$$

مثال $\Rightarrow$ $\omega^* = 0.1 , M^* = -40 , \varphi^* = -180$

$$y_{ss} = 10^{-2} \sin(0.1t - 180) = 0.01 \sin(0.1t - 180)$$

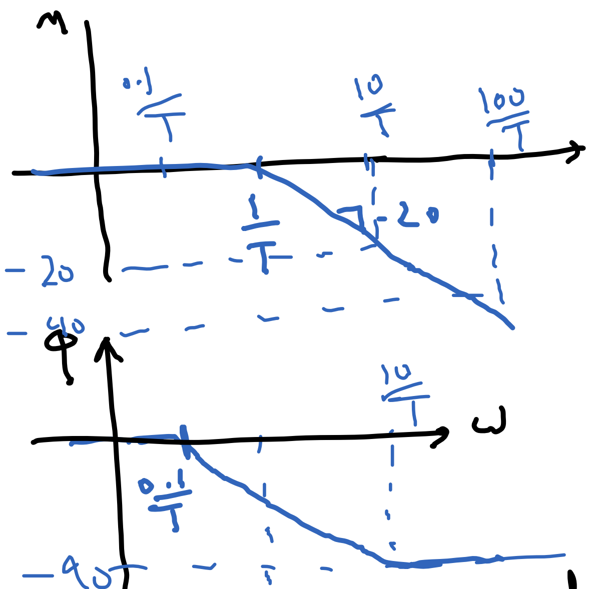
$$M^* = 0 \longrightarrow y_{ss} = 1 \sin(\omega^* t + \varphi^*)$$

$$M^* = 40 \longrightarrow y_{ss} = 100 \sin(\omega^* t + \varphi^*)$$

$$r(t) \rightarrow \boxed{\frac{1}{Ts+1}} \rightarrow y_{ss} = ?$$

$$|G(j\omega)| = 10^{M/20}$$

$$r(t) = \sin\left(\frac{0.1}{T}\right) + \sin\left(\frac{10}{T}\right) + \sin\left(\frac{100}{T}\right)$$



$$\Rightarrow y_{ss} = \sin\left(\frac{0.1}{T}\right) + 0.1\sin\left(\frac{10}{T}t - 90\right)$$

$$+ 0.01\sin\left(\frac{100t}{T} - 90\right)$$

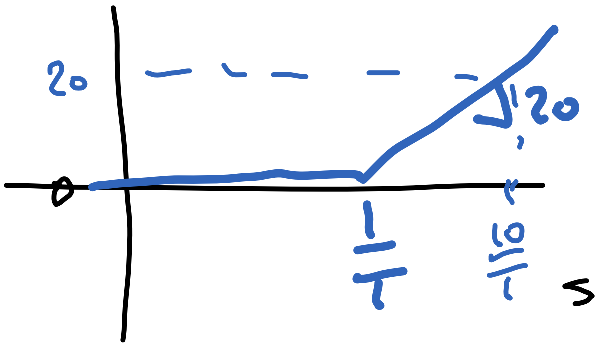
$$\Rightarrow y_{ss} \simeq \sin\left(\frac{0.1}{T}t\right)$$

این تابع تبدیل ورودی‌های فرکانس بالا را فیلتر می‌کند. به همین دلیل
به این تابع تبدیل (low Pass Filter) فیلتر پایین‌گذر

$$G(s) = Ts + 1$$

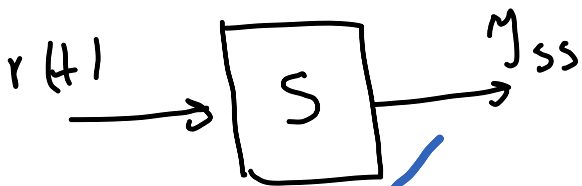
high pass filter فیلتر بالاگذر

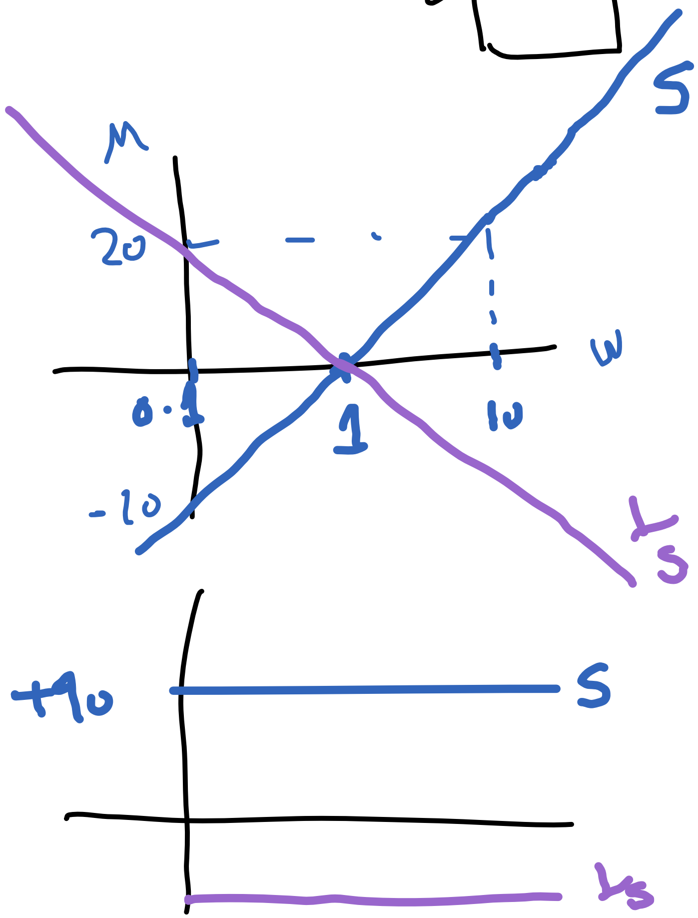
فرکانسهای پایین عبور می‌دهد. فرکانسهای بالا را نیز تقویت می‌کند.



$$\sin\left(\frac{10t}{T}\right) \longrightarrow \boxed{Ts+1} \longrightarrow y_{ss} = 10\sin\left(\frac{10t}{T} + 90\right)$$

$$\sin\left(\frac{100t}{T}\right) \longrightarrow y_{ss} = 100\sin\left(\frac{100t}{T} + 90\right)$$





عامل مشتق گیر، سیگنالهای فرکانس بالا را تقویت می کند

$$r(t) = \sin(10t) + \sin(100t)$$

$$y_{ss} = 10\sin(10t+90) + 100\sin(100t+90)$$

انتگرال گیر برعکس عمل می کند، سیگنالهای فرکانس بالا را حذف می کند

سیستم های مینیمم فاز (MF) و غیر مینیمم فاز (NMF)

توابع تبدیلی که در نیم راست صفحه S هیچ صفر و قطبی ندارند، MF گفته می شوند و آنهایی که حداقل یک صفر یا قطب در سمت راست دارند NMF هستند.

$$G_1(S) = \frac{S+1}{(S+2)(S+3)} \quad , \quad G_2(S) = \frac{S-1}{(S+2)(S+3)}$$

$G_1 \rightarrow MF$

$$G_3(S) = \frac{S+1}{(S-2)(S+3)}$$

$G_2 , G_3 \rightarrow$ غیر مینیمم فاز هستند NMF

تشخیص MF بودن یک سیستم از روی دیاگرام بود،

$$G_1(s) = \frac{T_1 s + 1}{T_2 s + 1} \quad , \quad G_2(s) = \frac{-T_1 s + 1}{T_2 s + 1} \quad , \quad T_2 , T_1 > 0$$

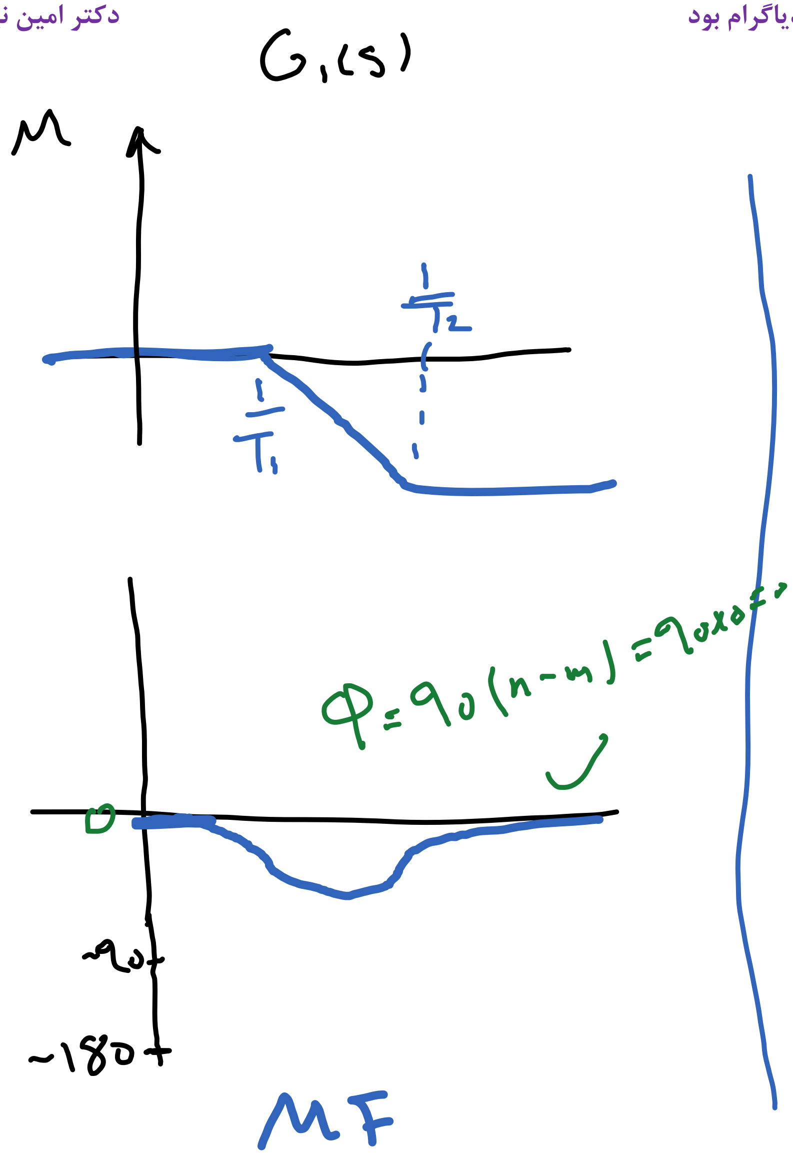
نمودار دامنه G_1 و G_2 یکسان هست

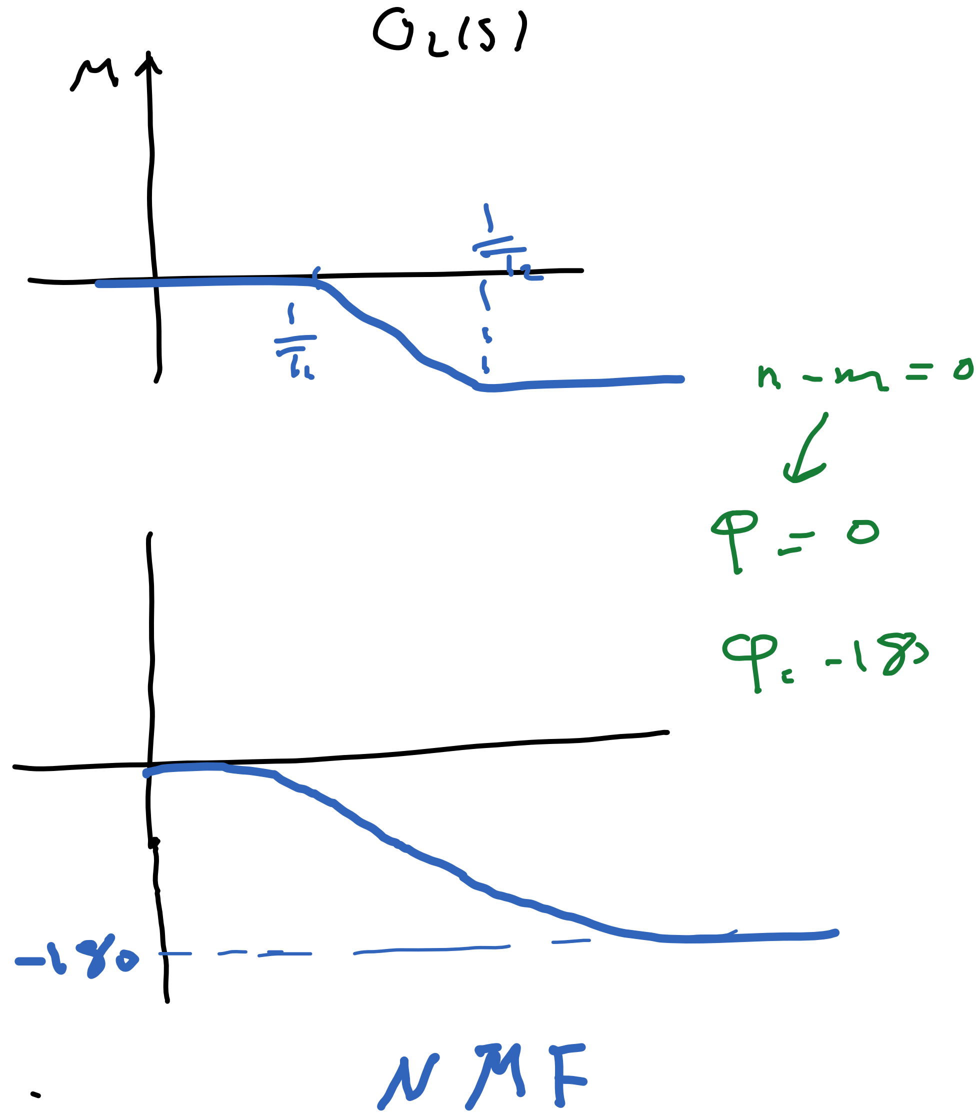
$$|G_1(j\omega)| = |G_2(j\omega)| = \frac{\sqrt{T_1^2\omega^2+1}}{\sqrt{T_2^2\omega^2+1}}$$

$$\varphi_1 = \angle G_1(j\omega) = tg^{-1} T_1\omega - tg^{-1} T_2\omega$$

$$\varphi_2 = \angle G_2(j\omega) = tg^{-1}(-T_1\omega) - tg^{-1} T_2\omega = -tg^{-1} T_1\omega - tg^{-1} T_2\omega$$

$$\text{if } T_1 = T_2 \longrightarrow \varphi_1 = 0 , \quad \varphi_2 = -2 tg^{-1} T\omega$$


$G_1(s)$
M
$\frac{1}{T_1}$
$\frac{1}{T_2}$
0
-90
-180
$\varphi = 90(n-m) = 90 \times 0 = 0$
MF

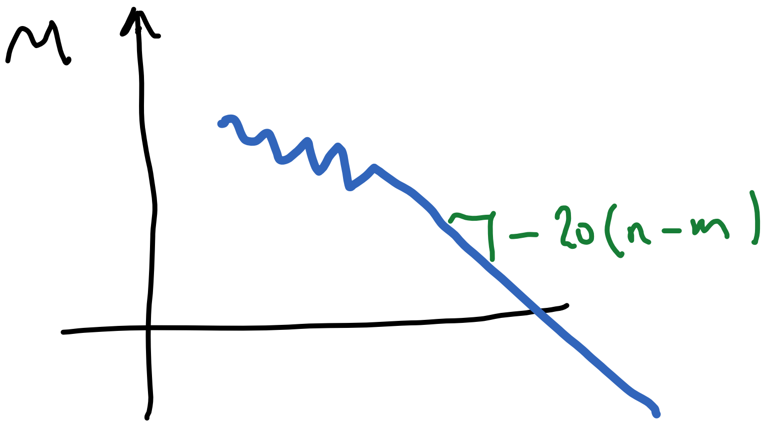

$G_2(s)$
M
$\frac{1}{T_1}$
$\frac{1}{T_2}$
$n - m = 0$
$\varphi = 0$
$\varphi = -180$
-180
NMF

قاعده کلی تشخیص MF بودن سیستم از روی دیاگرام بد

$$G(s) = \frac{s^m + b_1 s^{m-1} + \cdots}{a_0 s^n + a_1 s^{n-1} + \cdots} \Rightarrow G(j\omega) = \frac{(j\omega)^m + \cdots}{a_0 (j\omega)^n + \cdots}, \quad n \geq m$$

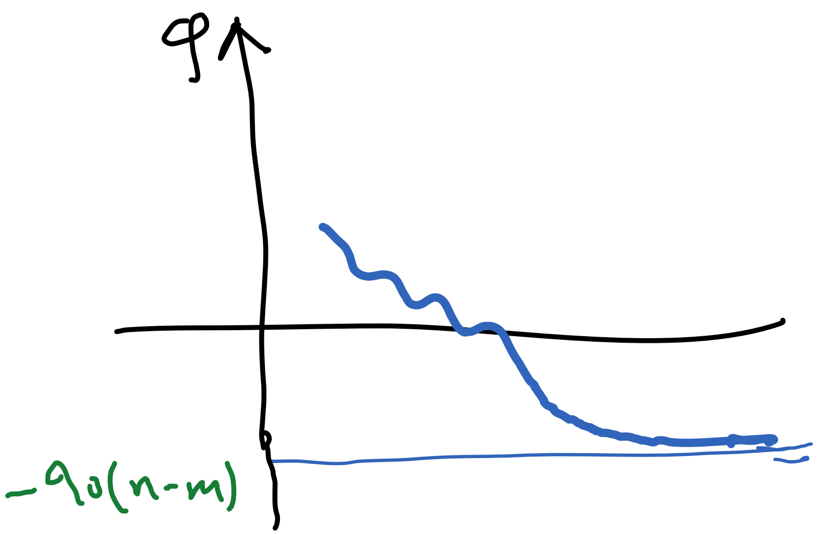
$$\left| G(j\omega) \right|_{\omega \to \infty} = \left| \frac{1}{a_0 \omega^{n-m}} \right| \Rightarrow M = 20 \log \left| G(j\omega) \right|_{\omega \to \infty} = -20(n-m) \log \omega$$

$$\varphi_{\omega \to \infty} = -(n-m) \times 90$$



برای یک سیستم MF

- شیب منحنی دامنه در $\omega=\infty$ $= -20\times(n-m)$
- زاویه فاز $= -90(n-m)$



برای سیستم NMF

- شیب منحنی دامنه $= -20(n-m)$
- زاویه فاز $\neq -90(n-m)$

$\omega \to \infty$

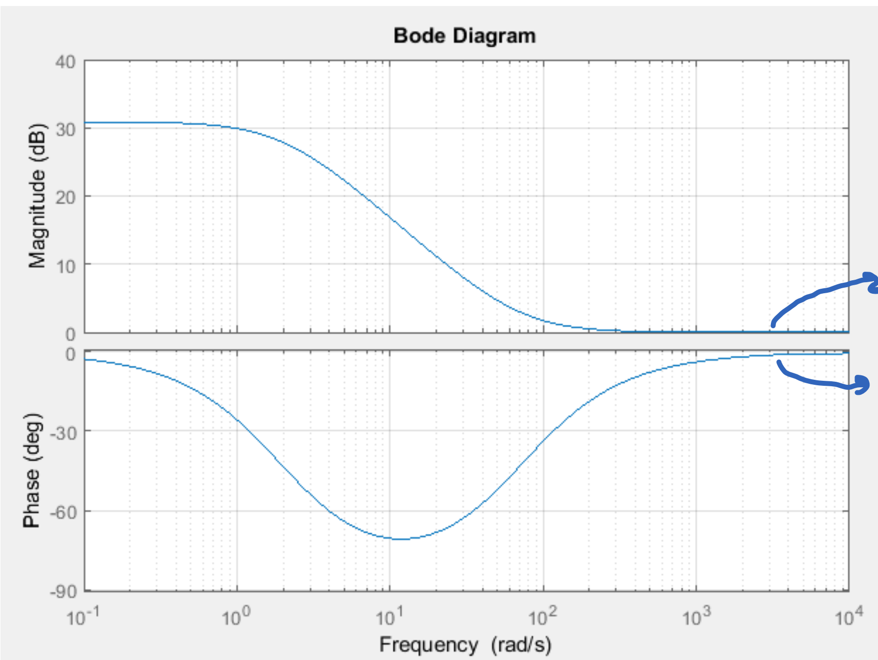
$$\frac{(s+70)}{(s+2)}$$

$$\frac{1}{(s+2)(s+50)}$$

$n - m = 2$

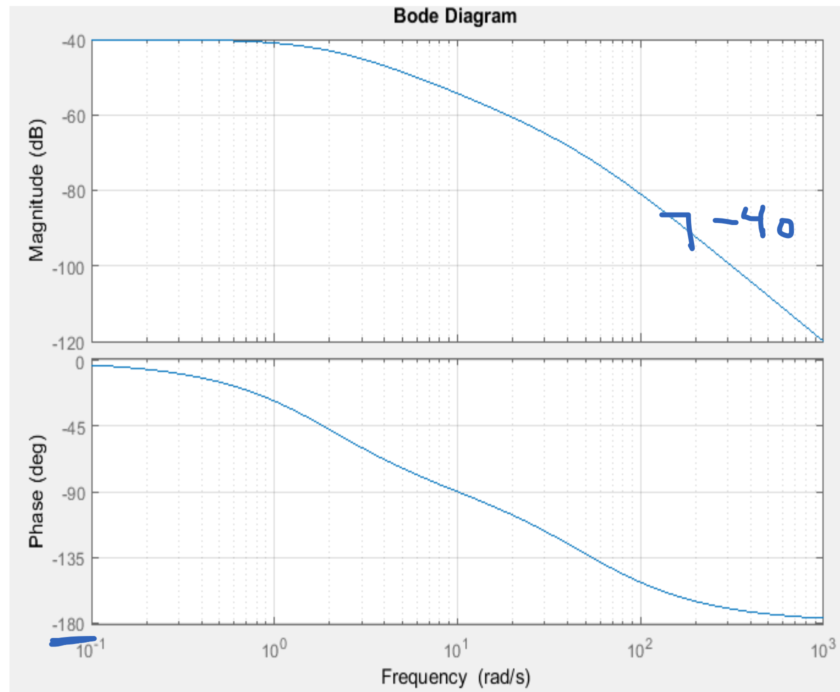
φ

$-40 = -20(n-m)$



$n - m = 0$

$-90 \times 0 = 0$



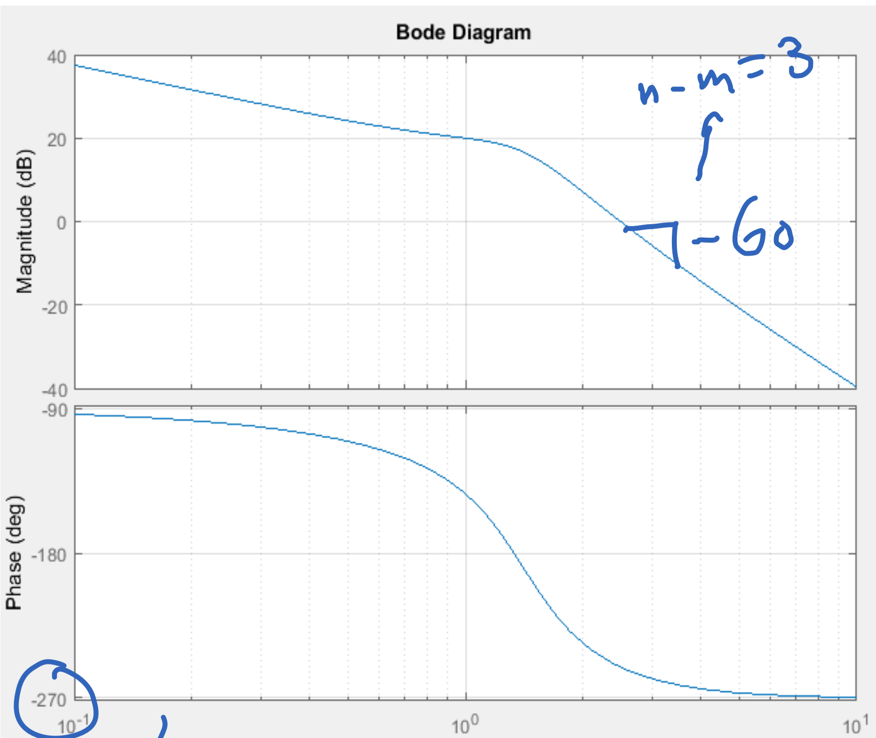
-40

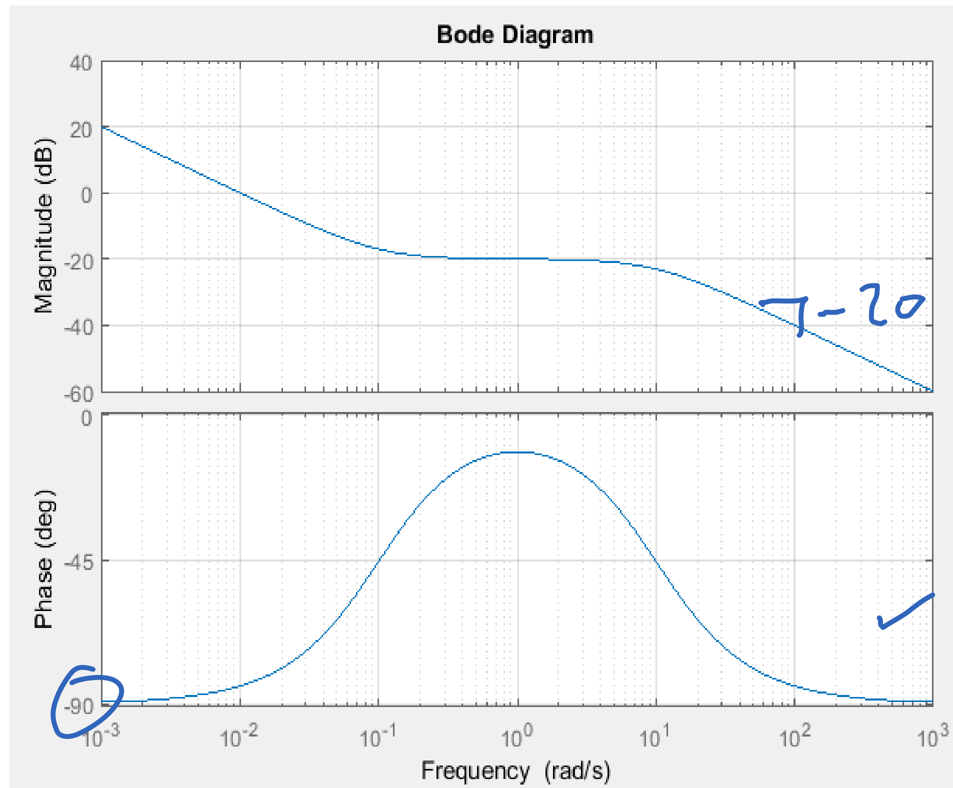
✓

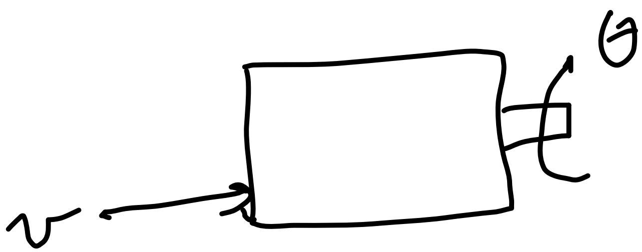
$\varphi = -90 \times (n-m) = -90 \times 2 = -180$

$$\frac{10\,(s+3)}{s\,(s+2)\,(s^2+s+2)} \qquad \begin{aligned} &\rightarrow m=1 \\ &\rightarrow n=4 \end{aligned}$$

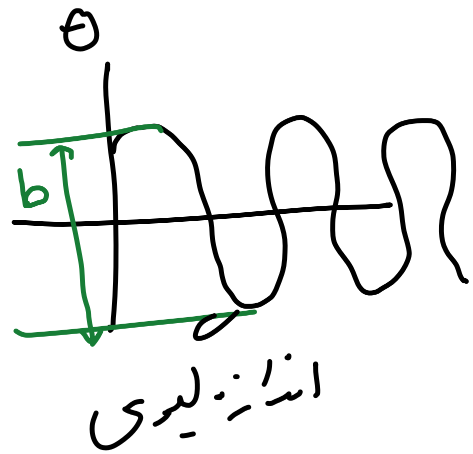
$$\frac{(s+0.1)}{s\,(s+10)}$$

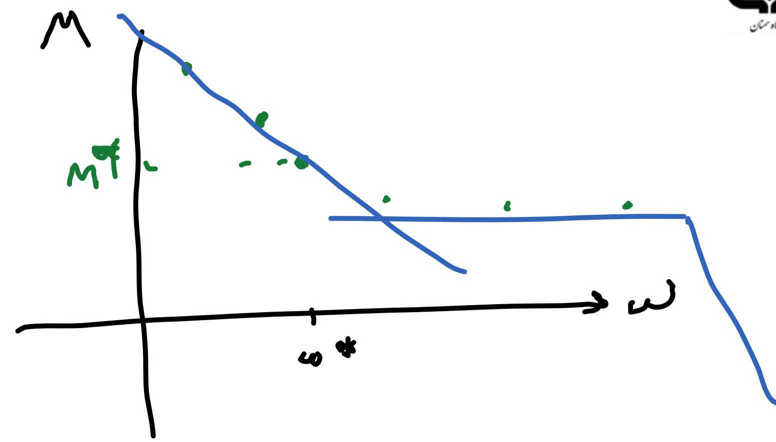


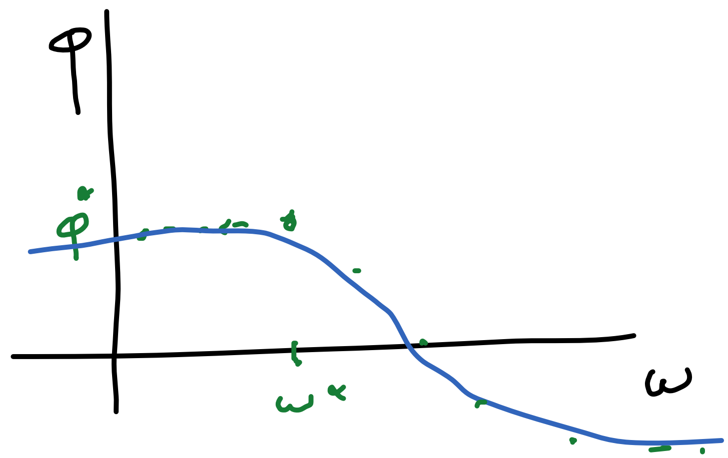




@ $\omega = \omega^*$







$$\frac{b}{a} = |G(j\omega^*)| \rightarrow 20\log|G(j\omega^*)| = M^*$$

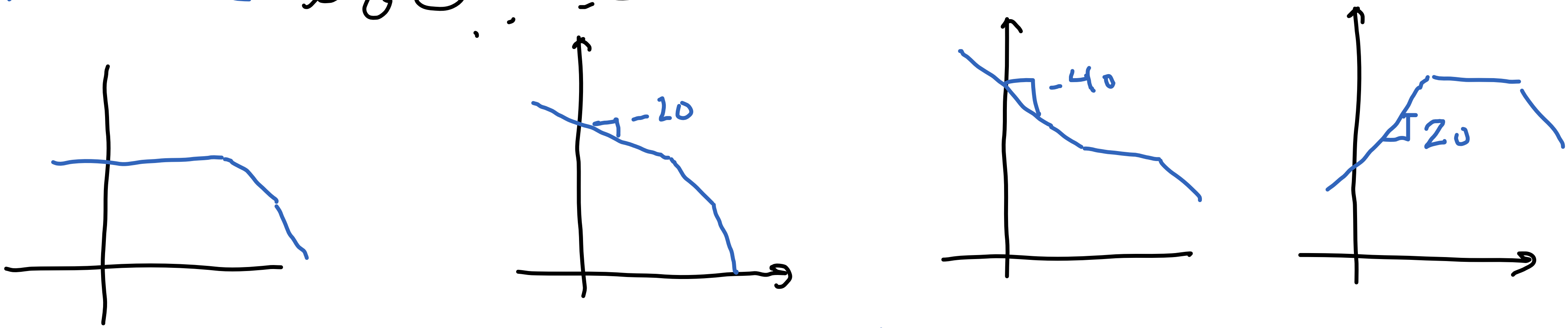
$$\omega \rightarrow 0.01,\ 0.05,\ 0.1,\ 1,\ 10,\ \ldots$$

یافتن تابع تبدیل از روی دیاگرام بود (از لحاظ عملی امکان‌پذیر است)

- تابع تبدیل یک سیستم MF را می‌توان به صورت تقریبی از روی دیاگرام بود به دست آورد.
- در ابتدا مجانب‌های منحنی‌های دامنه را رسم می‌کنیم. شیب مجانب‌ها مضربی از ± 20 (و همچنین نقاط شکست)
- اگر در ابتدا شیب منحنی دامنه $20\,dB/dec$ کاهش یابد یک عامل مرتبه یک $\dfrac{1}{\frac{s}{\omega_1}+1}$ داریم
- 〃 〃 〃 〃 〃 افزایش 〃 〃 $\dfrac{s}{\omega_1}+1$ داریم
- 〃 〃 〃 〃 〃 $40\,dB/dec$ کاهش یابد دو 〃 〃 〃 $\dfrac{1}{\left(\frac{s}{\omega_1}\right)^2+\frac{2\xi s}{\omega_1}+1}$ داریم

- اگر شیب منفی در منحنی $40 dB/decade$ افزایش پیدا کند یک عامل مرتبه 2 به $\left(\frac{s}{\omega_1}\right)^2 + \frac{2\xi s}{\omega_1} + 1$ داریم

- شیب اول نمودار، عامل انتگرال گیر یا مشتق گیر را تعیین می کند ← نوع Type

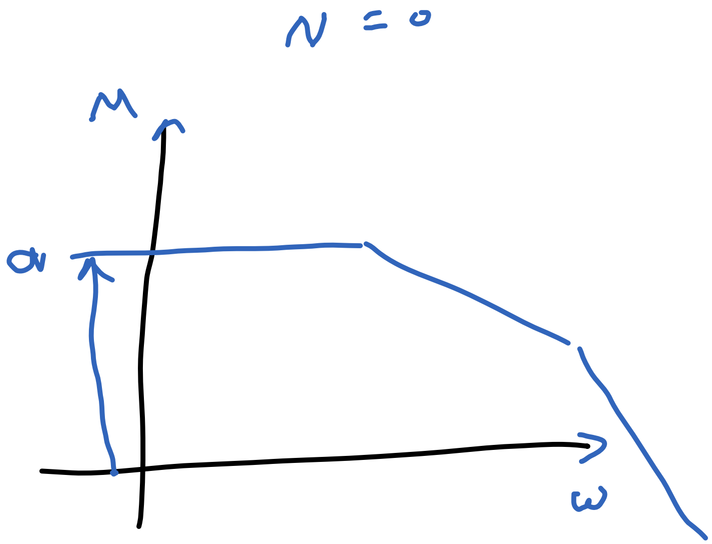


$G(s) = \frac{''}{s^0(--)}$ $N=0$

$G(s) = \frac{(\;)}{s^1(\;)}$ $N=1$

$G(s) = \frac{(\;)}{s^2(--)}$ $N=2$

$G(s) = \frac{s(\;)}{(\;)}$ $N=-1$



- تعیین بهره K

$$\lim_{\omega \to 0} M = a = 20\log|G(j\omega)| = 20\log K$$

$$\Rightarrow \log K = \frac{a}{20} \Rightarrow \boxed{K = 10^{a/20}}$$

$$\Rightarrow \lim_{\omega \to 0} G(j\omega) = \frac{K}{(j\omega)^N}$$

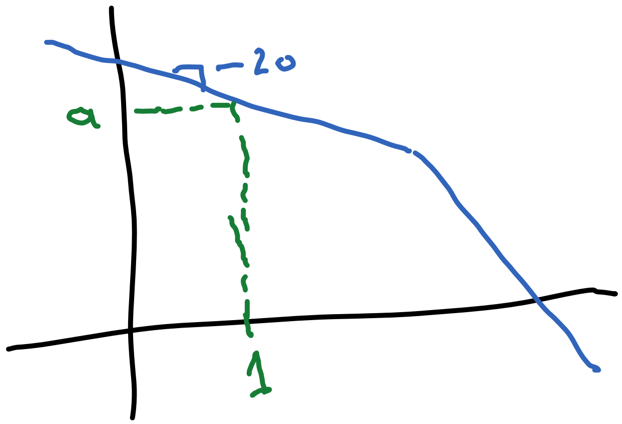
$$G(j\omega) = \frac{K\left(\frac{j\omega}{\omega_1}+1\right)(\cdots)\cdots}{(j\omega)^N\left(\frac{j\omega}{\omega_2}+1\right)(\quad)\cdots}$$

$$N = 0 \Rightarrow \lim_{\omega \to 0} |G(j\omega)| = K$$

$N = 1$

$$M = 20\log|G(j\omega)| = 20\log\left|\frac{K}{j\omega}\right|$$

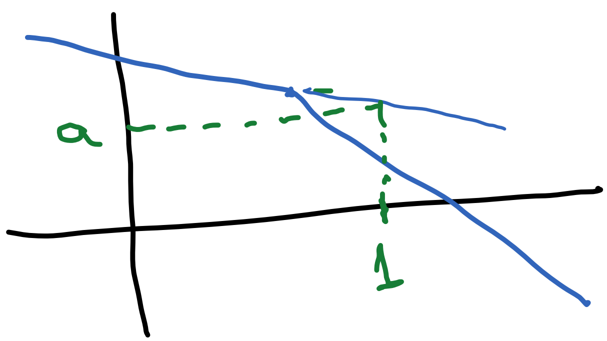
$\omega \to 0$

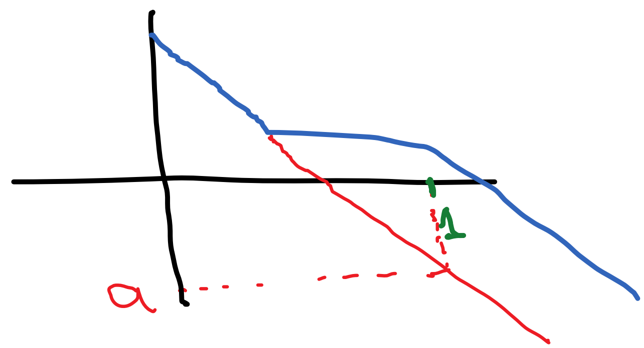


$$\omega = 1 \Rightarrow M = 20\log K = a$$

$$\Rightarrow \boxed{K = 10^{a/20}}$$

تقاطع فرکانس $\omega=1$ با امتداد قسمت اول نمودار

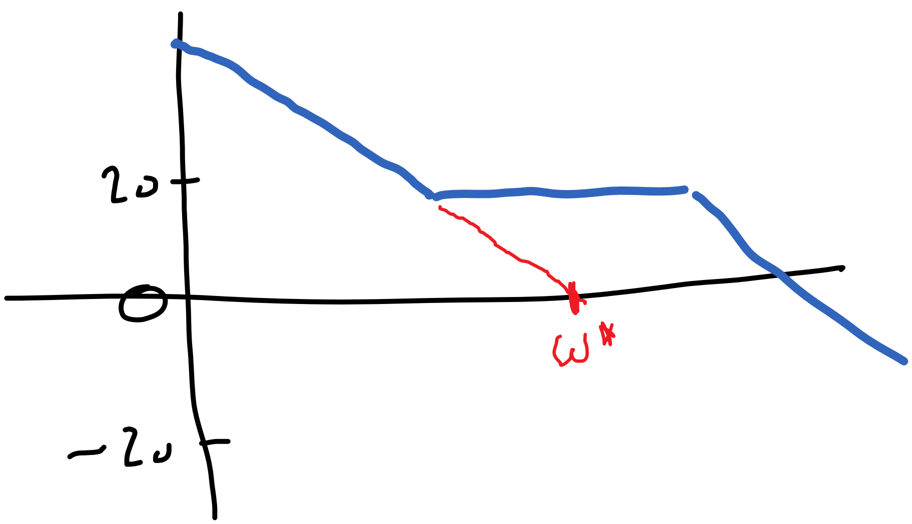




روش دوم
برای
یافتن K

$$M = 20\log\left|\frac{K}{j\omega}\right| = 20\log K - 20\log\omega = 0$$

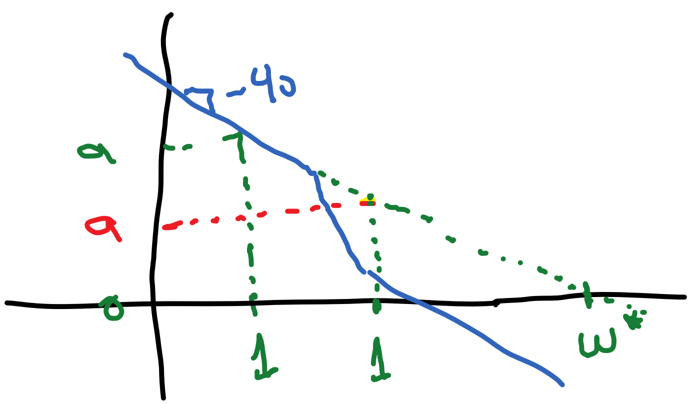
$$\Rightarrow \boxed{K = \omega^*}$$

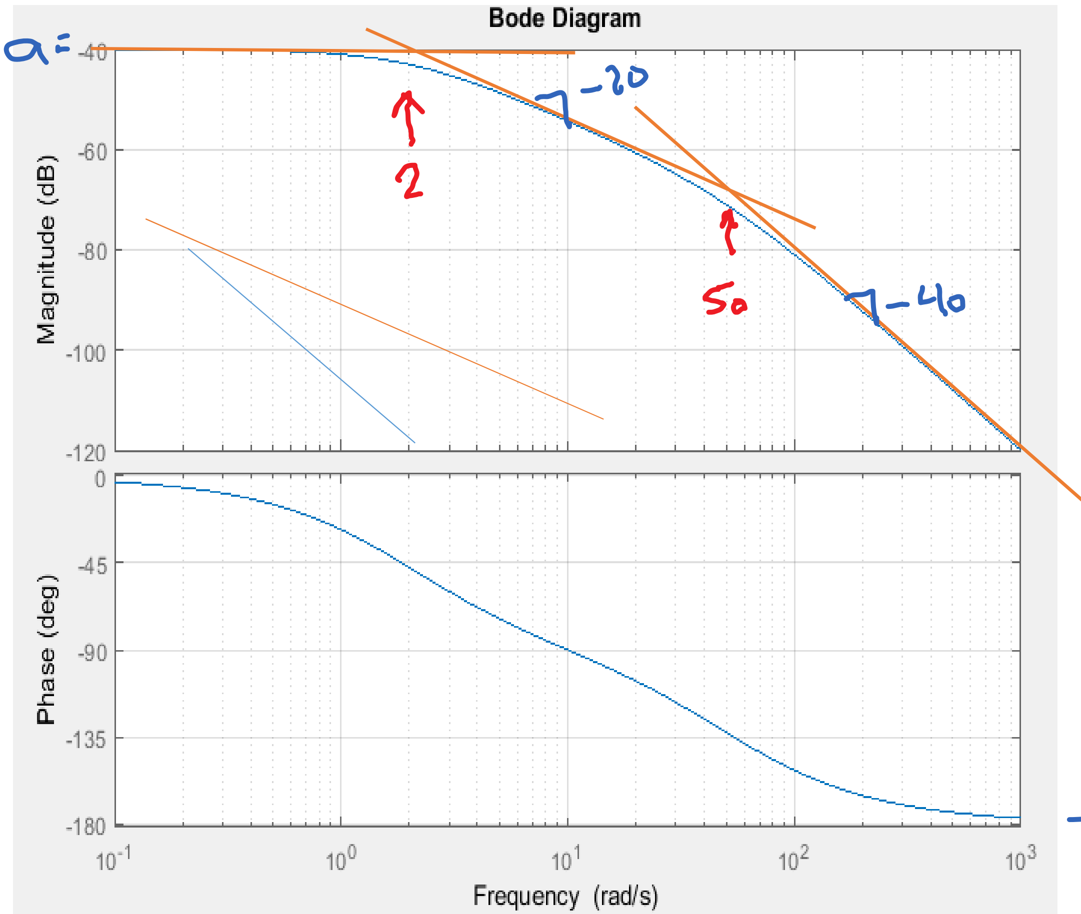


$N = 2$ ، $M = 20\log\left|\frac{K}{(j\omega)^2}\right|$

روش اول @ $\omega = 1 \rightarrow 20\log K = a \Rightarrow K = a^{10/20}$

روش دوم $M = 20\log K - 20\log\omega^2 = 0 \Rightarrow K = \omega^2 \Rightarrow \omega = \sqrt{K}$





MF

1

(s+2) (s+50)

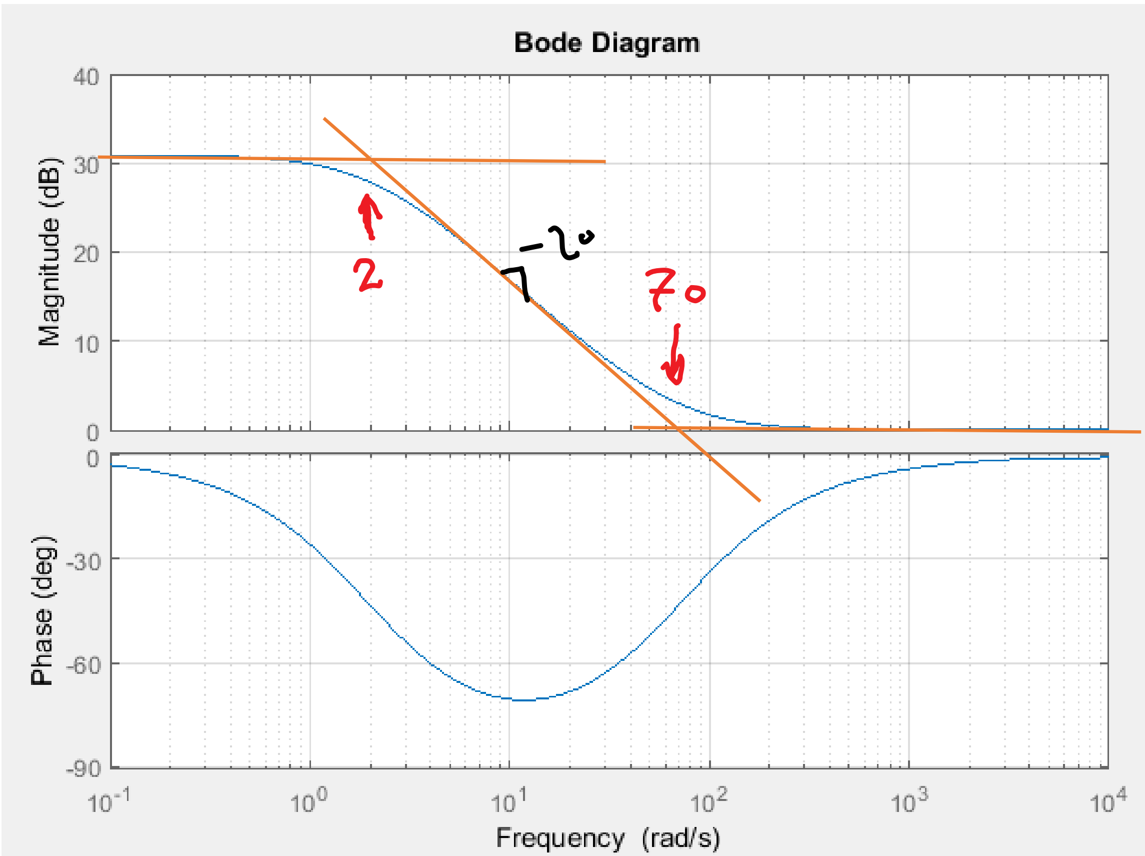
$$G(s) = \frac{0.01}{\left(\frac{s}{2}+1\right)\left(\frac{s}{50}+1\right)}$$

$$a=-40 \rightarrow K = 10^{a/20} = 10^{-40/20}$$

$$\Rightarrow K = 10^{-2} = 0.01$$

$-180 \rightarrow MF$

$$G(s) = \frac{1}{(s+2)(s+50)}$$



(s+70)

(s+2)

$$G(s) = \frac{\left(\frac{s}{70}+1\right)\times 40}{\left(\frac{s}{2}+1\right)}$$

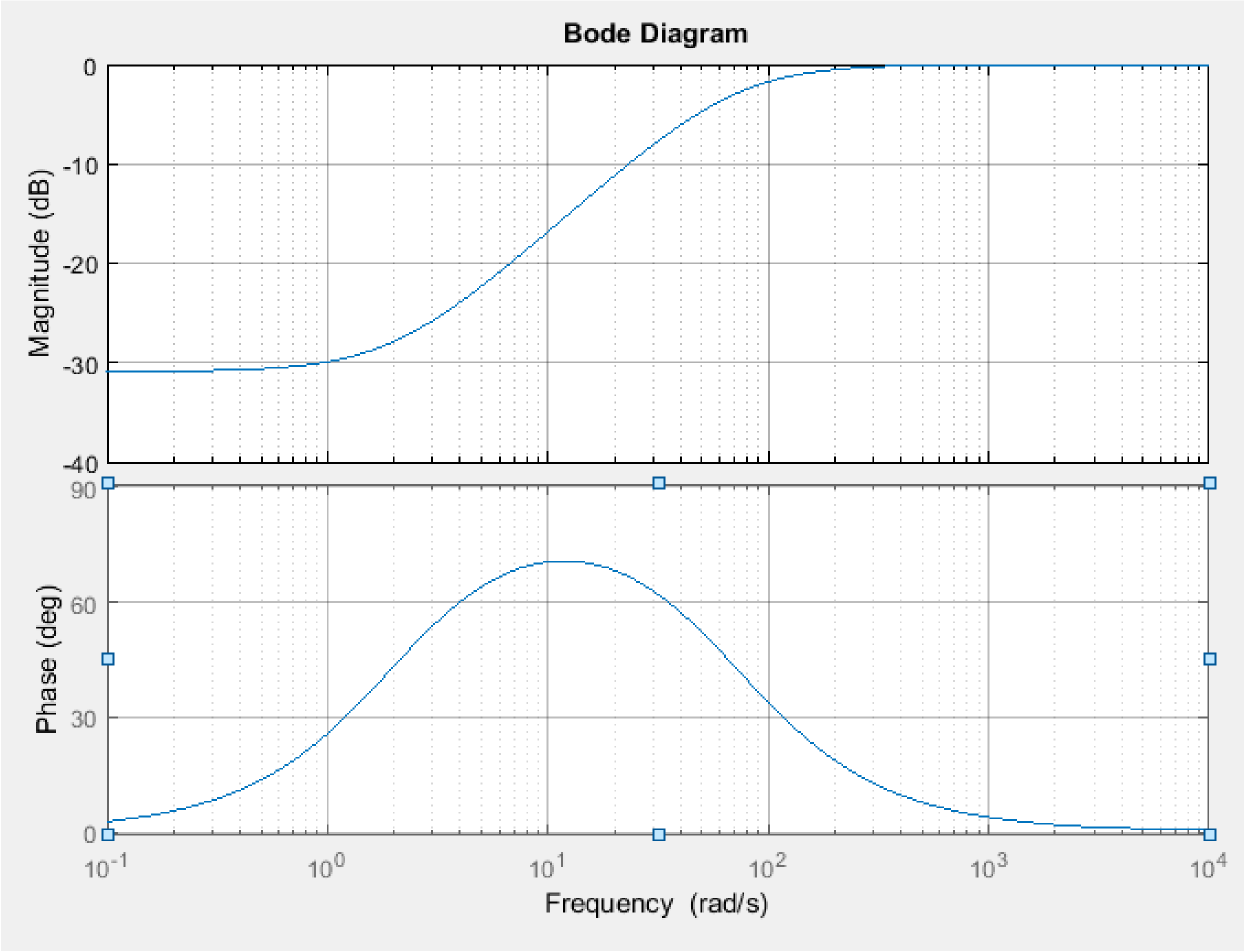
35

$$K = 10^{a/20} = 10^{32/20} = 40$$

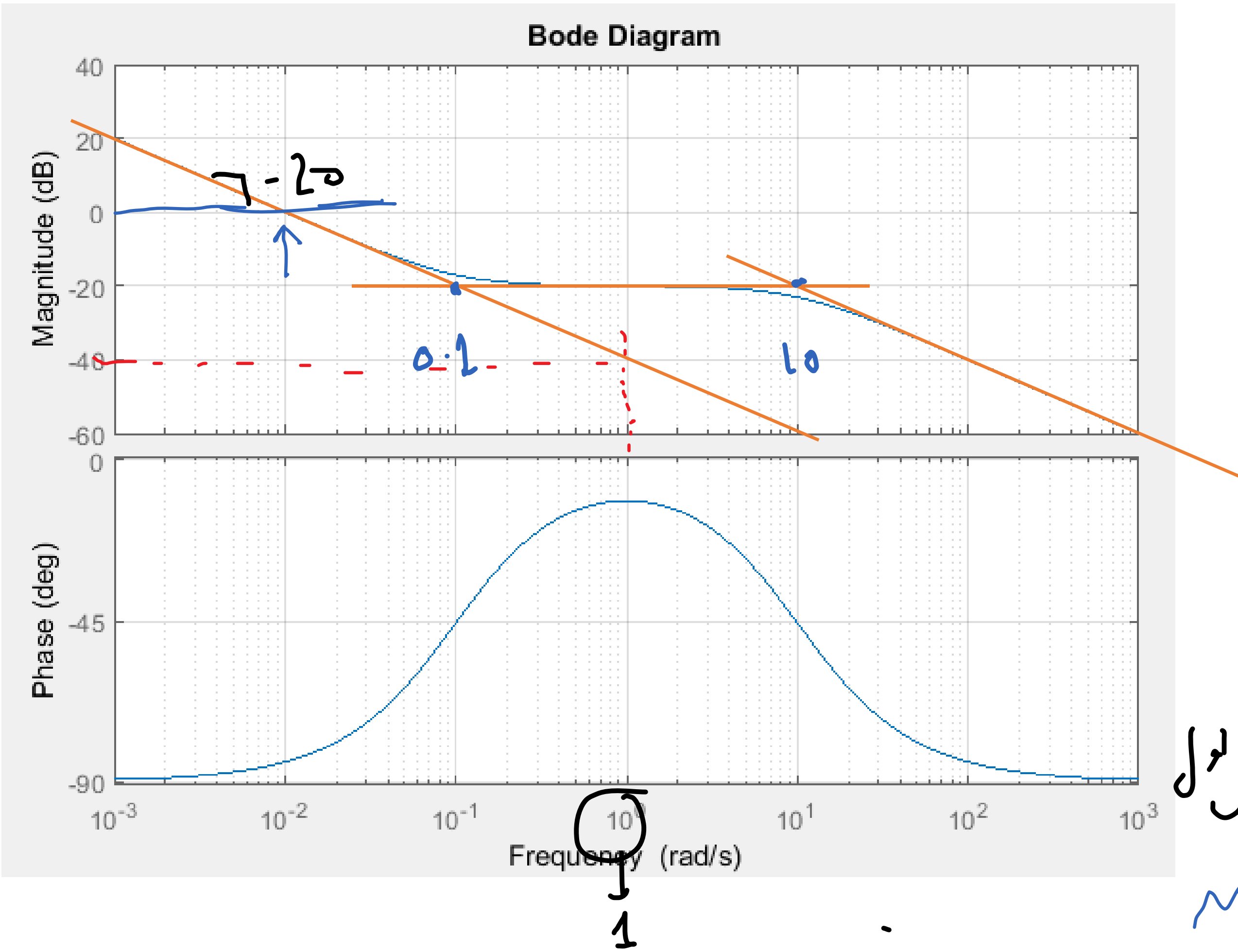
$$G(s) = \frac{(s+70)\times 40}{(s+2)\times 35}$$


(s+2)

(s+70)


Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
Phase (deg)
90
60
30
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Frequency (rad/s)

$$\frac{(s+0.1)}{s\,(s+10)}$$

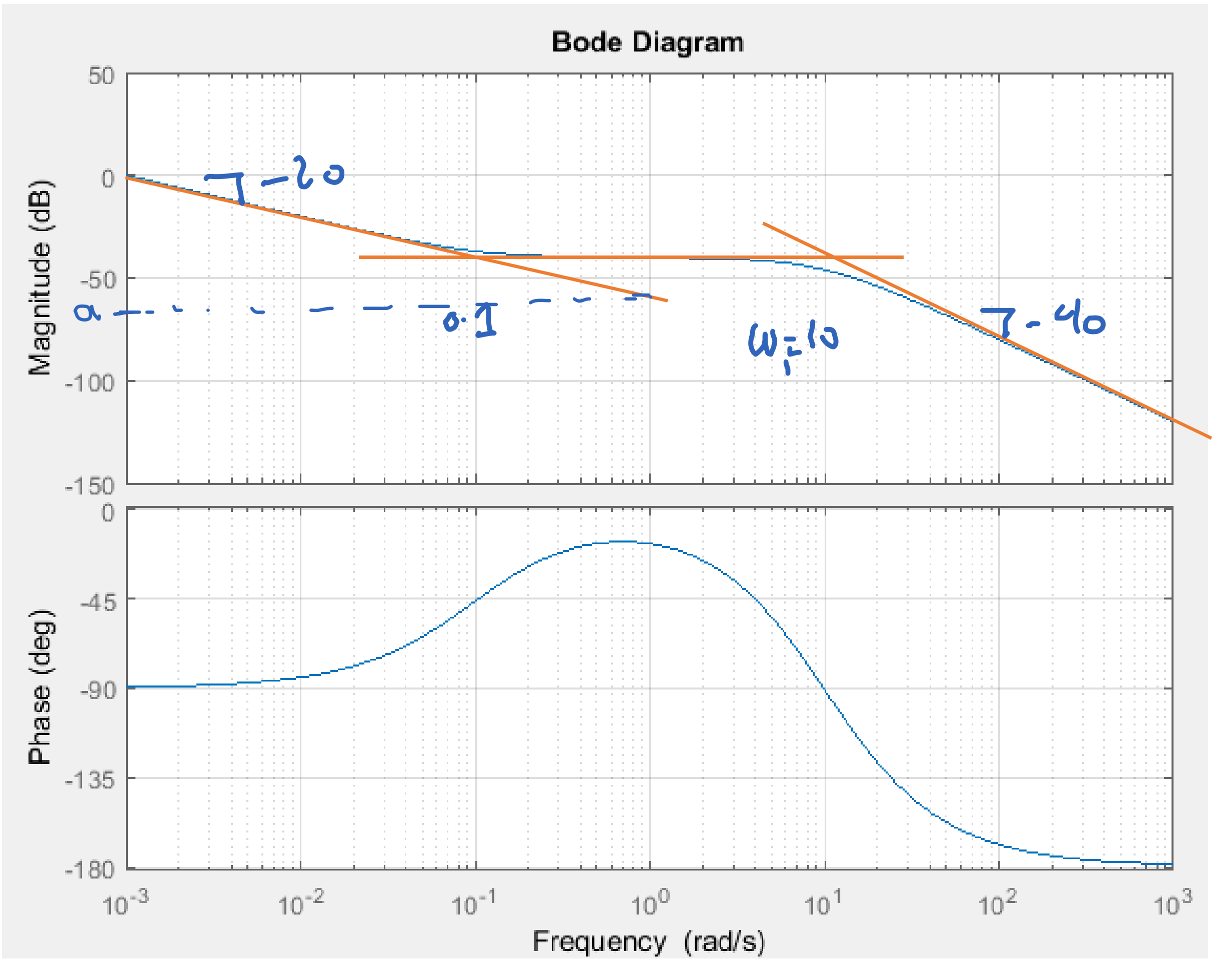


$$G(s)=\frac{\left(\frac{s}{0.1}+1\right)\times 0.01}{s\left(\frac{s}{10}+1\right)}$$

$$@\ \omega=1 \Rightarrow M\Big|_{\omega=1}=-40$$

روش اول $K=10^{a/20}=10^{-2}=0.01$

$M=0 \Rightarrow \quad K=\omega=10^{-2}$



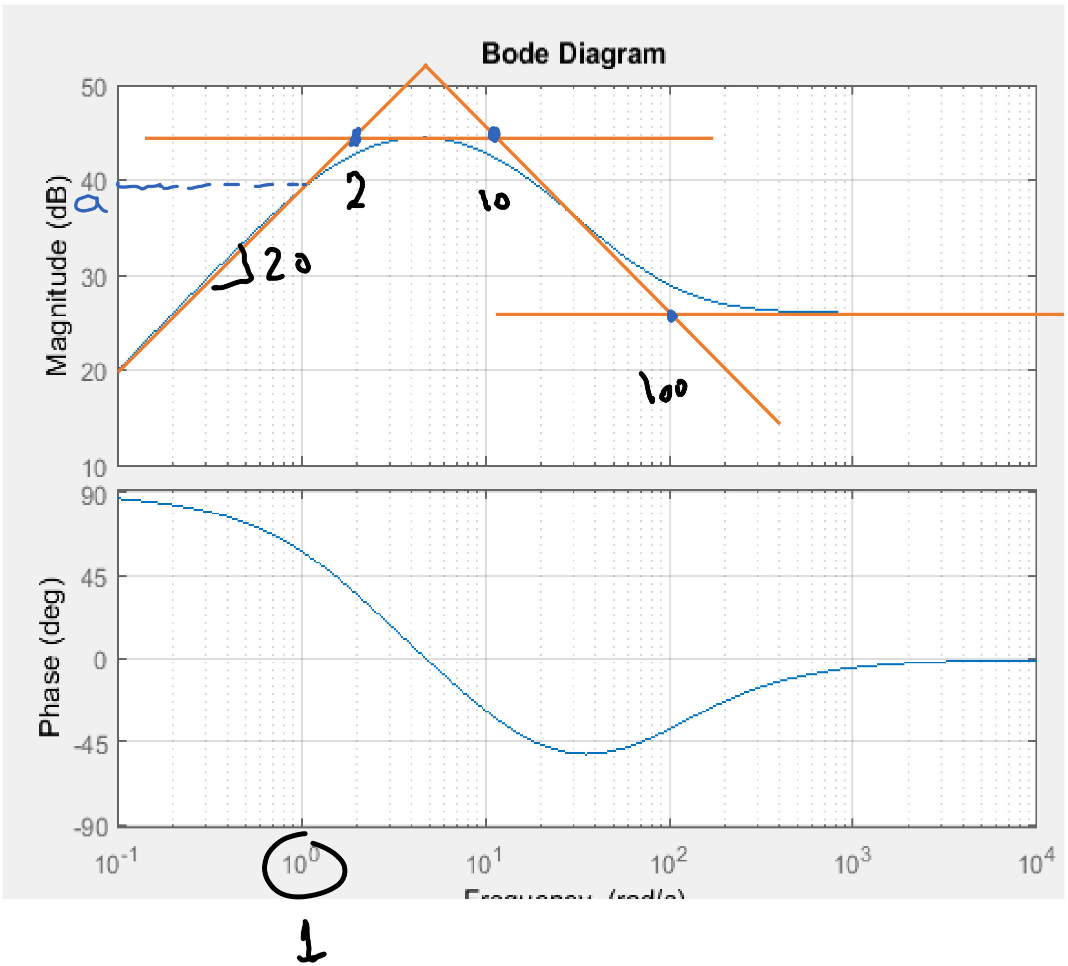
$$K = 10^{a/20}$$

$$\frac{(s+0.1)}{s\,(s+10)^{\wedge}2}$$

$$G_1(s) = \frac{\left(\frac{s}{0.1}+1\right)}{s\left(\frac{s}{10}+1\right)^2}$$

$$G_2(s) = \frac{\left(\frac{s}{0.1}+1\right)}{s\left(\left(\frac{s}{10}\right)^2+\frac{2\zeta s}{10}+1\right)}$$

$$\zeta = 0.2$$

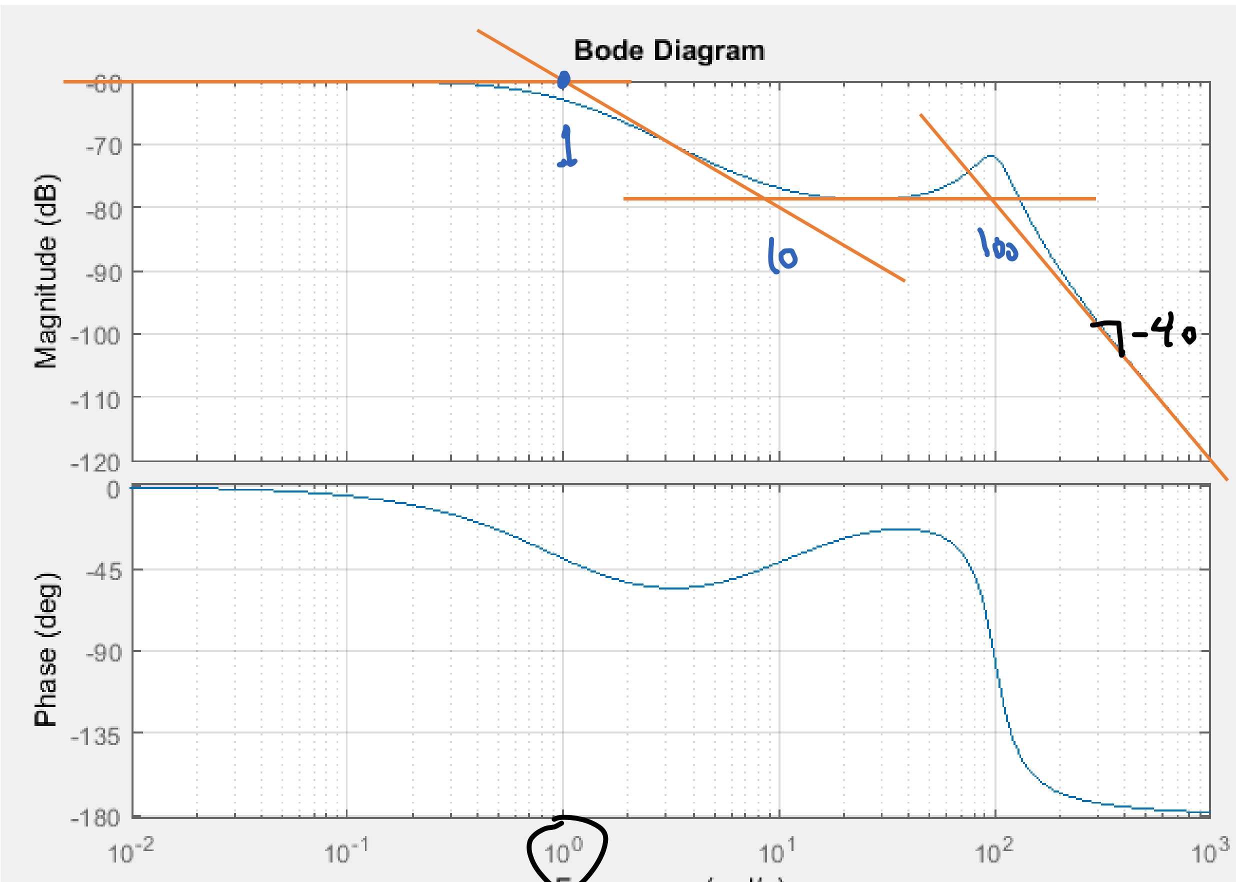


$$\frac{20\,s\,(s+100)}{(s+2)\,(s+10)}$$

$$G(s) = \frac{s\left(\frac{s}{100}+1\right)}{\left(\frac{s}{2}+1\right)\left(\frac{s}{10}+1\right)}$$

$$k = 10^{a/20} = 10^{2} = 100$$

$$G(s) = \frac{(s+10)}{(s+1)(s^2+40s+10^4)} = \frac{10}{10^4}\frac{\left(\frac{s}{10}+1\right)}{(s+1)\left(\frac{s^2}{10^4}+\frac{40s}{10^4}+1\right)}$$



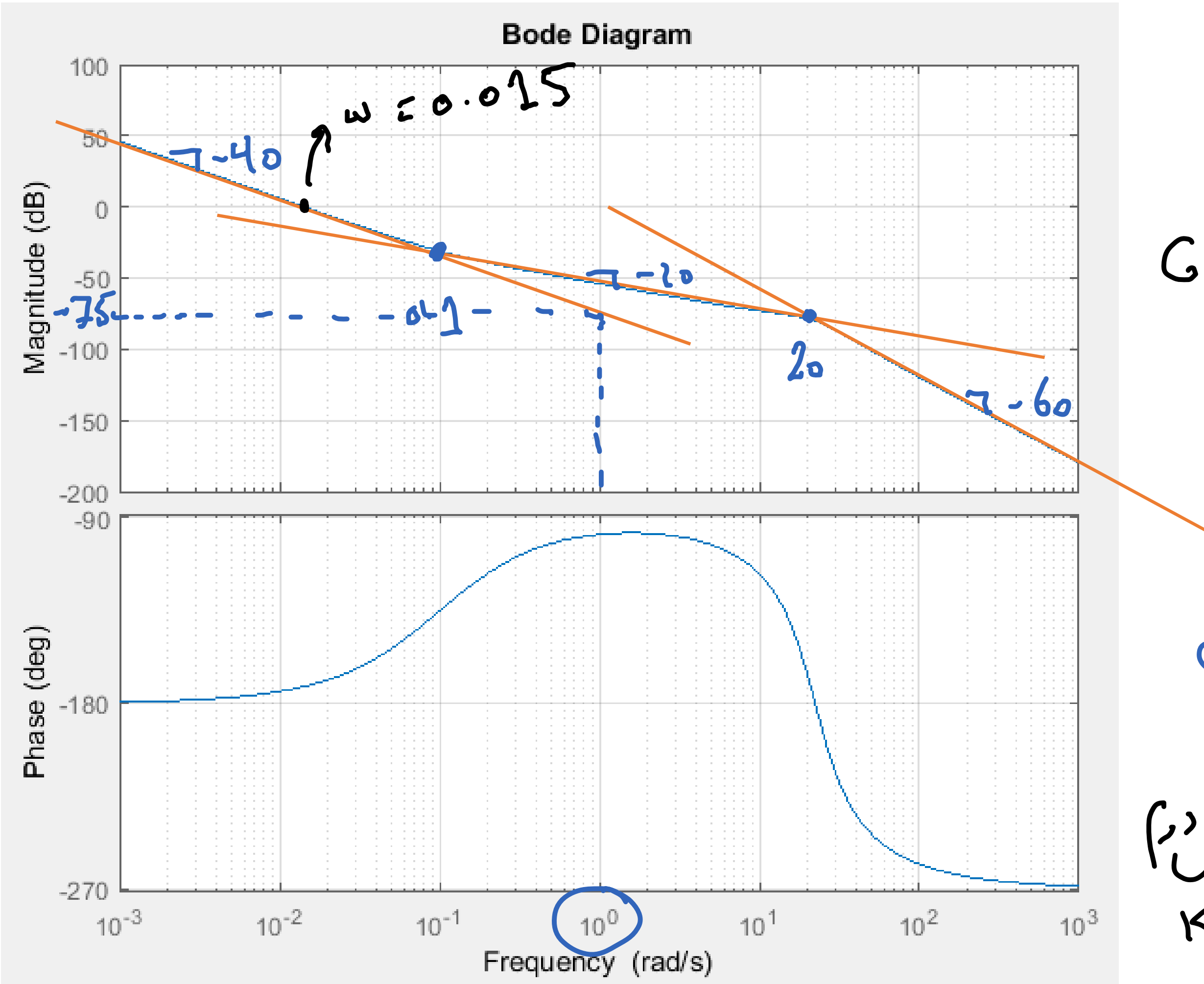
$$\frac{\left(\frac{s}{10}+1\right)10^{-4}}{(s+1)\left(\left(\frac{s}{100}\right)^2+\frac{2\zeta s}{100}+1\right)}$$

$\zeta = 0.2, 0.3$

$\log 2\zeta = -8 \rightarrow \zeta = $

$K = 10^{-80/20} = 10^{-4}$

$$\frac{(s+0.1)}{s^2\,(s^2 + 20s + 500)}$$



$$G(s) = \frac{\left(\frac{s}{0.1}+1\right)}{s^2\left(\left(\frac{s}{20}\right)^2 + \frac{2\xi s}{20} + 1\right)}$$

$$\xi = 0.5$$

$$a = -75 \rightarrow K = 10^{-\frac{75}{20}} = \frac{1}{5600} \rightarrow 0.00018$$

روش دوم K

$$K = \omega^2 = (0.015)^2 = 0.0002$$

کنترل اتوماتیک

تحلیل پاسخ فرکانسی

دیاگرام بود، بخش چهارم، خطای حالت ماندگار

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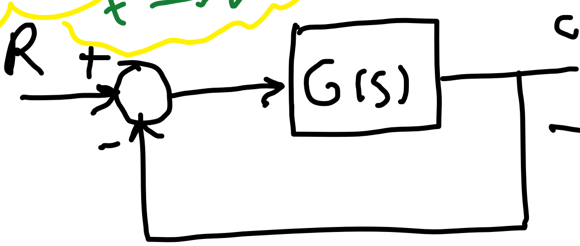
محاسبه خطای حالت ماندگار از روی دیاگرام بود

$e(t) = r(t) - c(t)$

$e_{ss} = \lim_{t \to \infty} e(t)$

یادآوری:



فیدبک واحد

$N=0 \rightarrow K_p = \lim_{s \to 0} G(s) = K$

ثابت خطای استاتیکی موقعیت

به ورودی پله $\rightarrow e_{ss} = \frac{1}{K_p + 1}$

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N (T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

Type

$N=1 \rightarrow K_v = \lim s G(s)$

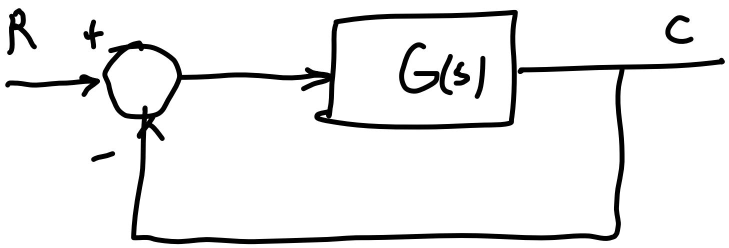
ثابت خطای استاتیکی سرعت

به ورودی شیب $e_{ss} = \frac{1}{K_v}$

$N=2 \rightarrow K_a = \lim s^2 G(s)$

به ورودی سهمی $e_{ss} = \frac{1}{K_a}$

محاسبه خطای حالت ماندگار را از روی دیاگرام بود:



فرض کنید دیاگرام بود را برای تابع تبدیل $G(s)$ داریم

- دیاگرام بود برای تابع تبدیل حلقه باز معلوم است.

می خواهیم خطای حالت ماندگار سیستم حلقه بسته را

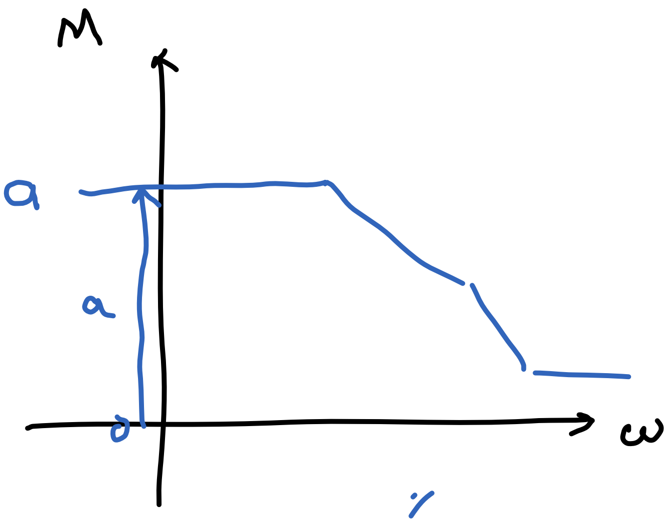
به ورودی های مختلف بدست آوریم

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)\cdots(T_m s + 1)}{s^N(T_1 s + 1)(T_2 s + 1)\cdots(T_p s + 1)}$$

$$G(j\omega) = \frac{K(T_a j\omega + 1)(T_b j\omega + 1)\cdots(T_m j\omega + 1)}{(j\omega)^N(T_1 j\omega + 1)(T_2 j\omega + 1)\cdots(T_p j\omega + 1)}$$

سیستم نوع صفر

$N = 0$



دیاگرام بود تابع $G(s)$

$$K_p = \lim_{s \to 0} G(s)$$

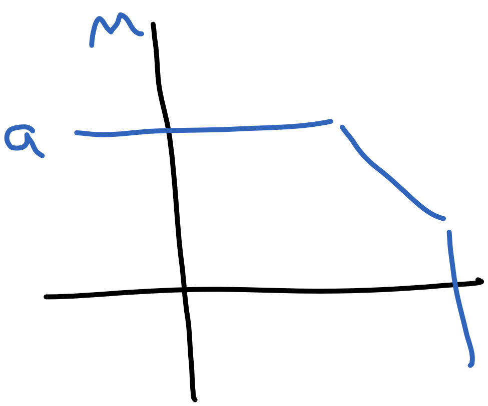
$$\Rightarrow K_p = \lim_{\omega \to 0} G(j\omega) = K$$

$$G(j\omega) = \frac{K(T_a j\omega + 1)(T_b j\omega + 1)\cdots(T_m j\omega + 1)}{(j\omega)^N (T_1 j\omega + 1)(T_2 j\omega + 1)\cdots(T_p j\omega + 1)}$$

$$M = \lim_{\omega \to 0} 20\log|G(j\omega)| = a = 20\log K_p$$

$$\Rightarrow K_p = 10^{a/20}$$

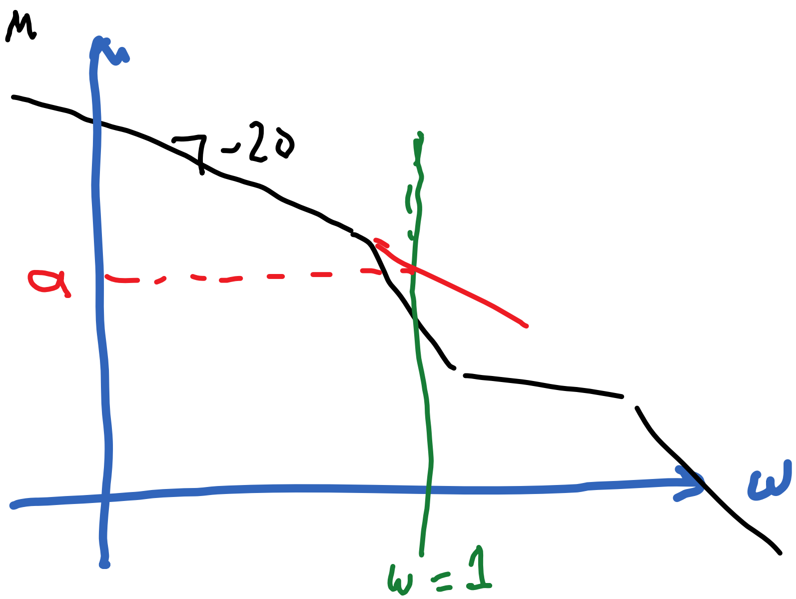
مراحل محاسبه خطای حالت ماندگار



- تشخیص نوع سیستم ← $N=0$
- تعیین مقدار دامنه در قسمت پایین از روی دیاگرام بود ← a
- محاسبه K_p ← $K_p = 10^{a/20}$
- محاسبه e_{ss} ←

ورودی پله: $e_{ss} = \dfrac{1}{K_p+1}$

شیب و سهمی " " $e_{ss} = \infty$



سیستم نوع $N=1$

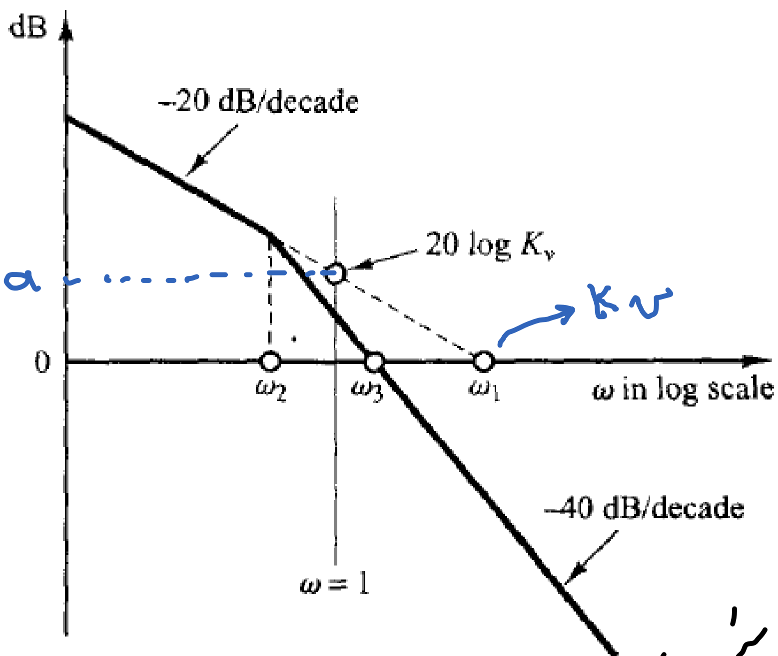
$$K_v = \lim_{s \to 0} s\,G(s)$$

$$K_v = \lim_{\omega \to 0} (j\omega)\,G(j\omega)$$

$$\Rightarrow \left|G(j\omega)\right|_{\omega \to 0} = \left|\frac{K_v}{j\omega}\right| \Rightarrow 20\log\left|G(j\omega)\right| = 20\log\left|\frac{K_v}{\omega}\right|$$

روش اول @ $\omega = 1 \Rightarrow 20\log K_v = 20\log\left|G(j\omega)\right|_{\omega \to 0} = a$

$$\Rightarrow \boxed{K_v = 10^{a/20}}$$



$$20\log|G(j\omega)| = 20\log\left|\frac{K_v}{\omega}\right| = 0$$

روش دوم $M = 0 \Rightarrow \boxed{K_v = \omega_1}$

$$\boxed{K_v = 10^{a/20}}$$

ورودی شیب $e_{ss} = \frac{1}{K_v}$

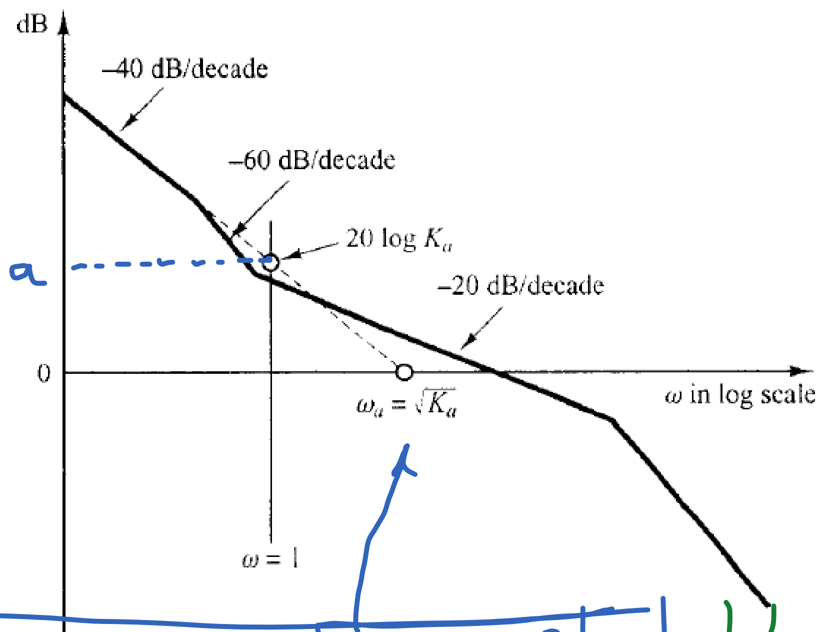
پله $e_{ss} = 0$ و سهمی $e_{ss} = \infty$

سیستم نوع 2 $\quad N=2$

$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{\omega\to 0} (j\omega)^2 G(j\omega)$$

$$\Rightarrow |G(j\omega)| = \left|\frac{K_a}{(j\omega)^2}\right|$$

$$\Rightarrow 20\log|G(j\omega)| = 20\log\left|\frac{K_a}{(j\omega)^2}\right|$$



روش اول: $\omega = 1 \Rightarrow 20\log K_a = 20\log|G(j\omega)|_{\omega=1} = a$

$$\Rightarrow \boxed{K_a = 10^{a/20}}$$

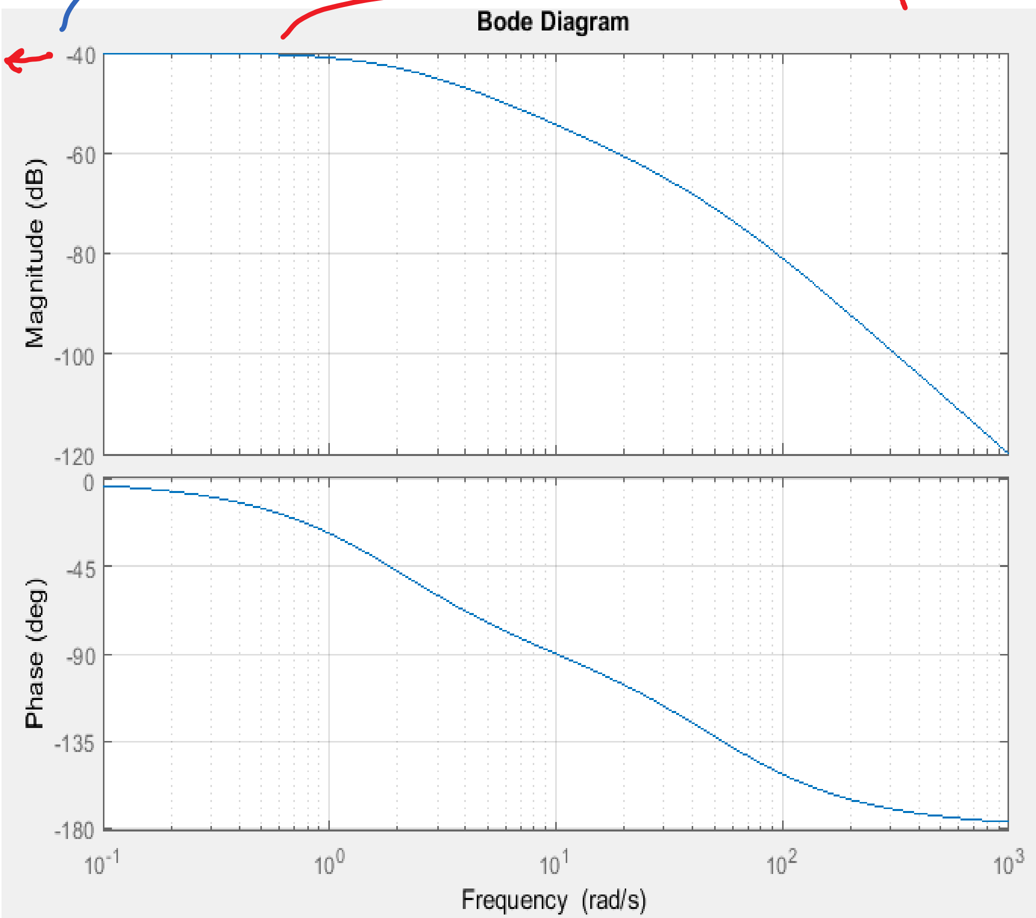
روش دوم: $M = 0 \Rightarrow \boxed{K_a = \omega_a^2}$

سهمی $e_{ss} = \frac{1}{K_a}$ → شیب $e_{ss} = 0$ ، پله $e_{ss} = 0$

$M = 20\log|G(j\omega)|$

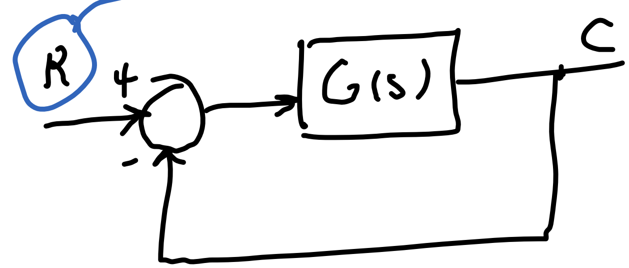
a

$N = 0 \longrightarrow K_p = ?$



$G(s)$

برای سیستم حلقه بسته زیر که دیاگرام بود تابع تبدیل حلقه باز آن $(G(s))$ به صورت روبرو داده شده است، مقدار خطای حالت ماندگار را به ورودی پله، شیب و سهمی به دست آورید:



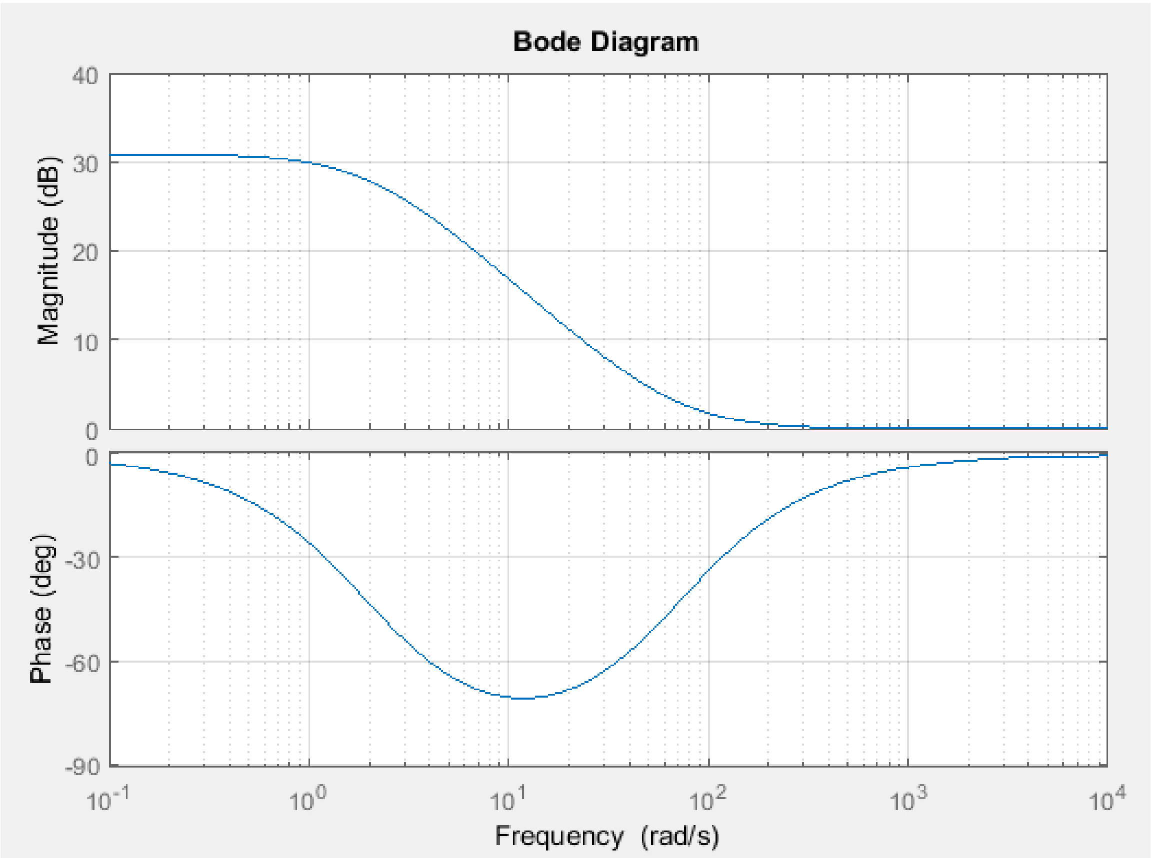
$$e_{ss} = \lim_{t \to \infty} \left(r(t) - c(t) \right)$$

$$K_p = 10^{a/20} = 10^{-40/20} = 10^{-2} = 0.01$$

به ورودی پله $e_{ss} = \dfrac{1}{K_p+1} = \dfrac{1}{1.01} \simeq 1$

" " شیب $e_{ss} = \infty$

" " سهمی $e_{ss} = \infty$



$N = 0$

$a = 32$

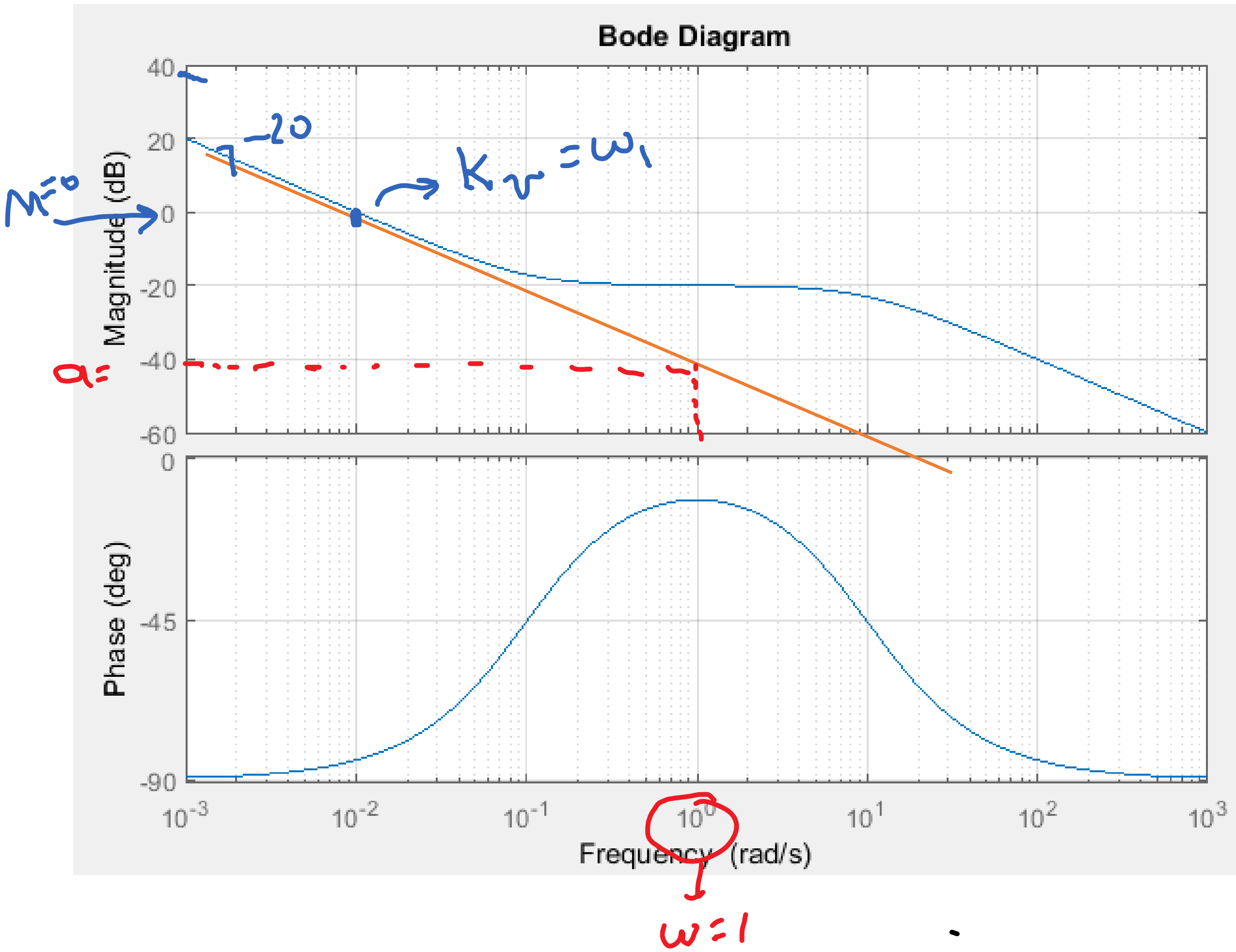
$K_p = 10^{32/20} = 10^{1.6} = 40$

پله $e_{ss} = \frac{1}{40+1} = \frac{1}{41}$

شیب $e_{ss} = \infty$

$\longrightarrow K = 10$

$N = 1 \longrightarrow K_v$



روش اول $\longleftarrow \omega = 1$

$$K_v = 10^{a/20} = 10^{-\frac{40}{20}} = 0.01$$

روش دوم $\longleftarrow M = 0$

$$K_v = \omega_1 = 10^{-2}$$

شیب $e_{ss} = \dfrac{1}{K_v} = \dfrac{1}{0.01} = 100$

پله $e_{ss} = 0$ ، سهمی $e_{ss} = \infty$

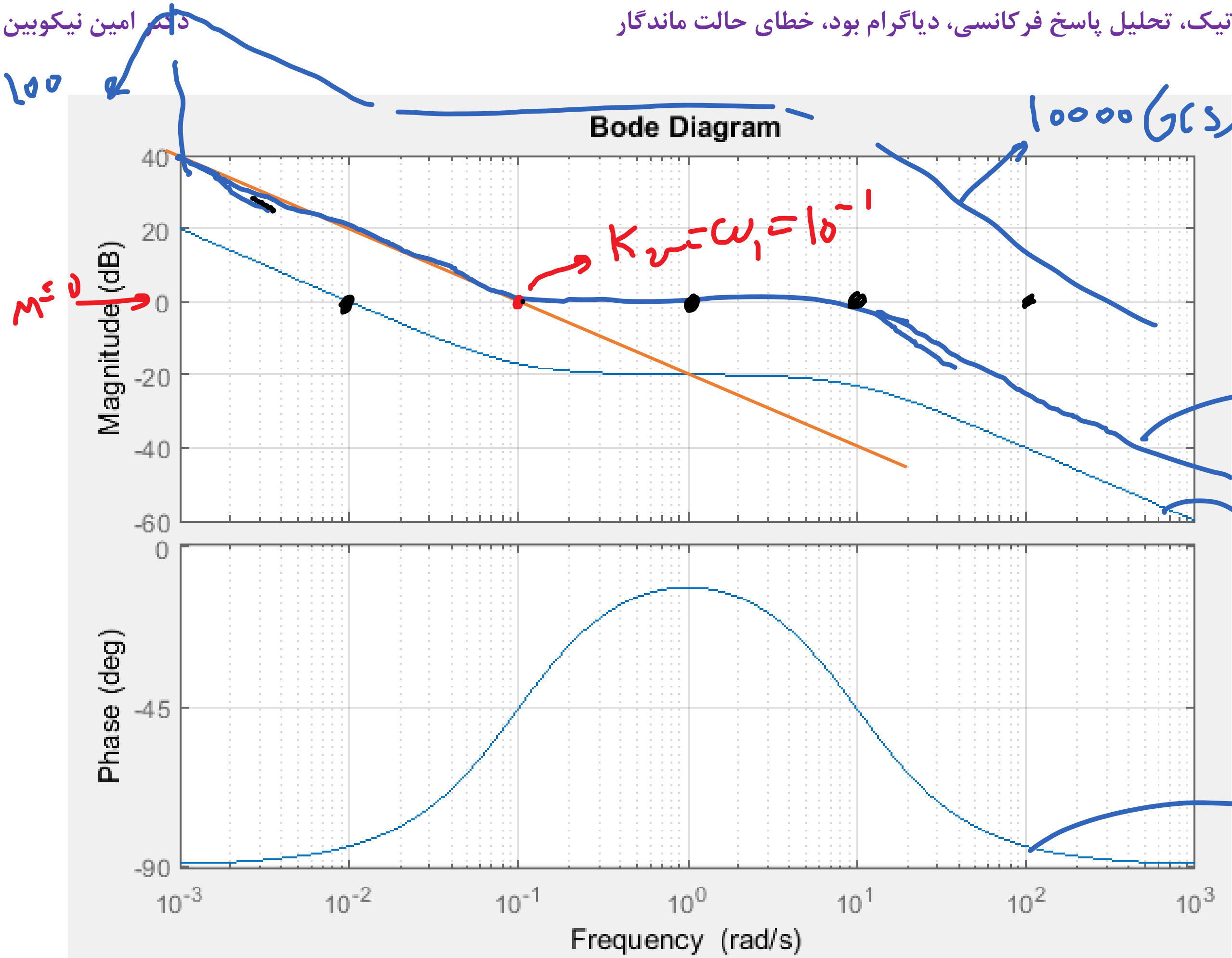
اگر ضریب بهره تابع تبدیل $G(s)$ را ۱۰ برابر کنیم، خطای حالت ماندگار چقدر خواهد شد

$$G(s) \longrightarrow G_1(s) = 10\,G(s)$$

دیاگرام دامنه $G_1(s)$ به اندازه $20\log 10$ به سمت بالا شیفت خواهد یافت

$$20\log|G_1(s)| = 20\log 10 + 20\log|G(s)| = 20 + M$$

مثبت



$$e_{ss} = \frac{1}{K_v} = \frac{1}{0.1} = 10$$

به شیب

اگر بخواهیم خطای حالت
ماندگار به ورودی شیب 0.01

$e_{ss} = 10 \leftarrow G(s)\times 10$ شروع $\to 40$

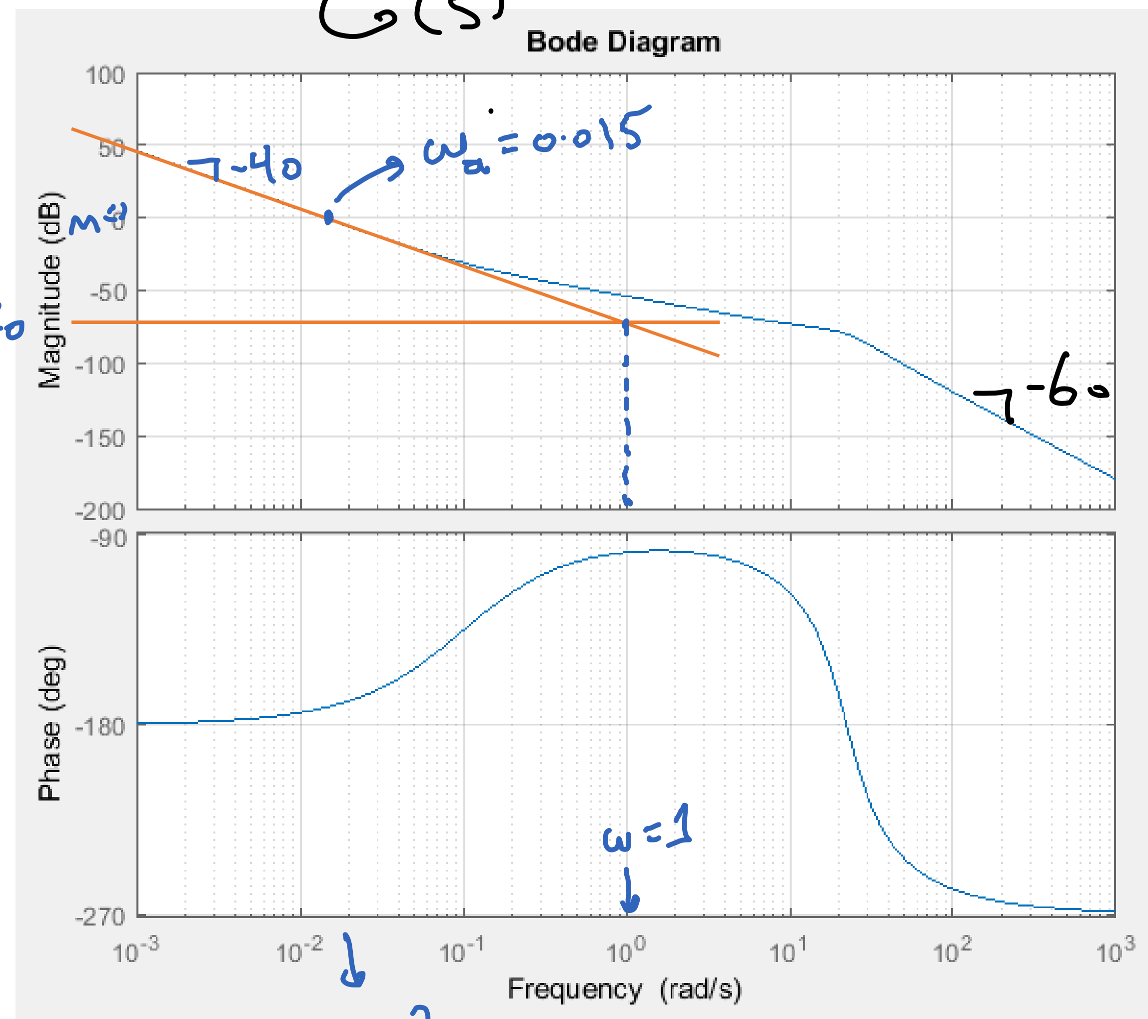
$e_{ss} = 1 \leftarrow G(s)\times 10 \times 10 \to 60$

$e_{ss} = 0.1 \leftarrow G(s)\times 1000 \to 80$

$e_{ss} = 0.01 \leftarrow G(s)\times 10000 \to 100$

شروع نمودار در فرکانس 10^{-3} ← از 100 خواهد بود

$G(s)$



$$\frac{(s+0.1)}{s^2(s^2+20s+500)}$$

$N = 2$

$\rightarrow k_a$

① $\omega = 1$ روش اول

$a = 70 \Rightarrow K_a = 10^{-\frac{70}{20}} = 0.0003$

$M = 0$ روش دوم

$0.01 \sim 0.02 \rightarrow 0.015$

$K_a = \omega_a^2 = (0.015)^2 = 0.0002$

سهمی $e_{ss} = \frac{1}{0.0002}$ ، پله، شیب $e_{ss} = 0$