

Table of Fourier Transforms

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{ix\omega} d\omega$		$\hat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$	
1.	$\begin{cases} 1 & \text{if }  x  < a \\ 0 & \text{if }  x  > a \end{cases}$		$\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$
2.	$\begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$		$\frac{i(e^{-ib\omega} - e^{ia\omega})}{\sqrt{2\pi}\omega}$
3.	$\begin{cases} 1 - \frac{ x }{a} & \text{if }  x  < a \\ 0 & \text{if }  x  > a \end{cases} \quad a > 0$		$2\sqrt{\frac{2}{\pi}} \frac{\sin^2(\frac{a\omega}{2})}{a\omega^2}$
4.	$\begin{cases} x & \text{if }  x  < a \\ 0 & \text{if }  x  > a \end{cases} \quad a > 0$		$i\sqrt{\frac{2}{\pi}} \frac{a\omega \cos(a\omega) - \sin(a\omega)}{\omega^2}$
5.	$\begin{cases} \sin x & \text{if }  x  < \pi \\ 0 & \text{if }  x  > \pi \end{cases}$		$i\sqrt{\frac{2}{\pi}} \frac{\sin(\pi\omega)}{\omega^2 - 1}$
6.	$\begin{cases} \sin(ax) & \text{if }  x  < b \\ 0 & \text{if }  x  > b \end{cases} \quad a, b > 0$		$i\sqrt{\frac{2}{\pi}} \frac{\omega \cos(b\omega) \sin(ab) - a \cos(ab) \sin(b\omega)}{\omega^2 - a^2}$
7.	$\frac{1}{a^2 + x^2}, a > 0$		$\sqrt{\frac{\pi}{2}} \frac{e^{-a \omega }}{a}$
8.	$\frac{x}{a^2 + x^2}, a > 0$		$-i\sqrt{\frac{\pi}{2}} \operatorname{sgn} \omega e^{-a \omega }$
9.	$\sqrt{\frac{2}{\pi}} \frac{a}{1 + a^2x^2}, a > 0$		$e^{-\frac{ \omega }{a}}$
10.	$\frac{\sin ax}{x}, a > 0$		$\begin{cases} \sqrt{\frac{\pi}{2}} & \text{if }  \omega  < a \\ \frac{1}{2}\sqrt{\frac{\pi}{2}} & \text{if }  \omega  = a \\ 0 & \text{if }  \omega  > a \end{cases}$
11.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(\frac{1}{2}ax)}{ax^2}, a > 0$		$\begin{cases} 1 - \frac{ \omega }{a} & \text{if }  \omega  < a \\ 0 & \text{if }  \omega  > a \end{cases}$
12.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(ax) - \sin^2(\frac{1}{2}ax)}{ax^2}, a > 0$		$\begin{cases} 1 & \text{if }  x  < a \\ (-x + 2a)/a & \text{if } a < x < 2a \\ (x + 2a)/a & \text{if } a < x < 2a \\ 0 & \text{if }  x  > 2a \end{cases}$
13.	$e^{-a x }, a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$
14.	$\begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}, a > 0$		$\frac{1}{\sqrt{2\pi}} \frac{1}{a + i\omega}$
15.	$\begin{cases} 0 & \text{if } x > 0 \\ e^{ax} & \text{if } x < 0 \end{cases}, a > 0$		$\frac{1}{\sqrt{2\pi}} \frac{1}{a - i\omega}$
16.	$ x ^n e^{-a x }, a > 0, n > 0$		$\frac{\Gamma(n+1)}{\sqrt{2\pi}} \left( \frac{1}{(a - i\omega)^{1+n}} + \frac{1}{(a + i\omega)^{1+n}} \right)$

Table of Fourier Transforms (continued)

$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega) e^{ix\omega} d\omega$	$\hat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$
17. $e^{-\frac{a}{2}x^2}, a > 0$	$\frac{1}{\sqrt{a}} e^{-\frac{\omega^2}{2a}}$
18. $e^{-ax^2}, a > 0$	$\frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}}$
19. $xe^{-\frac{a}{2}x^2}, a > 0$	$\frac{-i\omega}{a^{3/2}} e^{-\frac{\omega^2}{2a}}$
20. $x^2 e^{-\frac{a}{2}x^2}, a > 0$	$\frac{a - \omega^2}{a^{5/2}} e^{-\frac{\omega^2}{2a}}$
21. $x^3 e^{-\frac{a}{2}x^2}, a > 0$	$\frac{-i\omega(3a - \omega^2)}{a^{7/2}} e^{-\frac{\omega^2}{2a}}$
22. $e^{-\frac{x^2}{2}} H_n(x),$ $H_n, n$ th Hermite polynomial	$(-1)^n i^n e^{-\frac{\omega^2}{2}} H_n(\omega)$
23. $J_0(x),$ Bessel function of order 0	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{1-\omega^2}} & \text{if }  \omega  < 1 \\ 0 & \text{if }  \omega  > 1 \end{cases}$
24. $J_n(x),$ Bessel function of order $n \geq 0$	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{(-i)^n}{\sqrt{1-\omega^2}} T_n(\omega) & \text{if }  \omega  < 1 \\ 0 & \text{if }  \omega  > 1 \end{cases}$
$T_n, \text{Chebyshev polynomial of degree } n.$	
<b>Special Transforms</b>	
25. $\mathcal{F}(\delta_0(x))(\omega) = \frac{1}{\sqrt{2\pi}}$	27. $\mathcal{F}\left(\sqrt{\frac{2}{\pi}} \frac{1}{x}\right)(\omega) = -i \operatorname{sgn} \omega$
26. $\mathcal{F}(\delta_0(x-a))(\omega) = \frac{1}{\sqrt{2\pi}} e^{-ia\omega}$	28. $\mathcal{F}(e^{iax})(\omega) = \sqrt{2\pi} \delta_0(\omega - a)$
<b>Operational Properties</b>	
29. $\mathcal{F}(af + bg)(\omega) = a\mathcal{F}(f) + b\mathcal{F}(g)$	36. $\mathcal{F}(fg)(\omega) = \mathcal{F}(f) * \mathcal{F}(g)(\omega)$
30. $\mathcal{F}(f')(\omega) = i\omega\mathcal{F}(f)(\omega)$	37. $\mathcal{F}(f(x-a))(\omega) = e^{-ia\omega} \mathcal{F}(f)(\omega)$
31. $\mathcal{F}(f'')(\omega) = -\omega^2\mathcal{F}(f)(\omega)$	38. $\mathcal{F}(e^{iax}f(x))(\omega) = \mathcal{F}(f)(\omega - a)$
32. $\mathcal{F}(f^{(n)})(\omega) = (i\omega)^n \mathcal{F}(f)(\omega)$	39. $\mathcal{F}(\cos(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a) + \mathcal{F}(f)(\omega+a)}{2}$
33. $\mathcal{F}(xf(x))(\omega) = i \frac{d}{d\omega} \mathcal{F}(f)(\omega)$	40. $\mathcal{F}(\sin(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a) - \mathcal{F}(f)(\omega+a)}{2i}$
34. $\mathcal{F}(x^n f(x))(\omega) = i^n \frac{d^n}{d\omega^n} \mathcal{F}(f)(\omega)$	41. $\mathcal{F}(f(ax))(\omega) = \frac{1}{ a } \mathcal{F}(f)\left(\frac{\omega}{a}\right), a \neq 0$
35. $\mathcal{F}(f * g)(\omega) = \mathcal{F}(f)(\omega)\mathcal{F}(g)(\omega)$	42. $f(x) = \mathcal{F}(\hat{f})(-x), \mathcal{F}(\mathcal{F}(f)) = f(-x)$

Table of Fourier Cosine Transforms

$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \mathcal{F}_c(f)(\omega) \cos \omega x \, d\omega,$ $0 < x < \infty$		$\mathcal{F}_c(f)(\omega) = \widehat{f}_c(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos \omega x \, dx,$ $0 \leq \omega < \infty$	
1.	$\begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$		$\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$
2.	$e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$
3.	$x e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a^2 - \omega^2}{(a^2 + \omega^2)^2}$
4.	$e^{-ax^2/2}, \quad a > 0$		$\frac{1}{\sqrt{a}} e^{-\omega^2/2a}$
5.	$\cos ax e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{a\omega^3 + 2a^3}{4a^4 + \omega^4}$
6.	$\sin ax e^{-ax}, \quad a > 0$		$\sqrt{\frac{2}{\pi}} \frac{2a^3 - a\omega^2}{4a^4 + \omega^4}$
7.	$\frac{a}{a^2 + x^2}, \quad a > 0$		$\sqrt{\frac{\pi}{2}} e^{-a\omega}$
8.	$x^p, \quad 0 < p < 1$		$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p) \cos(p\omega/2)}{\omega^p}$
9.	$\begin{cases} \cos x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$		$\frac{1}{\sqrt{2\pi}} \left[ \frac{\sin a(1-\omega)}{1-\omega} + \frac{\sin a(1+\omega)}{1+\omega} \right]$
<b>Operational Properties</b>			
10.	$\alpha f(x) + \beta g(x)$		$\alpha \mathcal{F}_c(f)(\omega) + \beta \mathcal{F}_c(g)(\omega)$
11.	$f(ax), \quad a > 0$		$\frac{1}{a} \widehat{f}_c\left(\frac{\omega}{a}\right)$
12.	$f'(x)$		$\omega \widehat{f}_s(\omega) - \sqrt{\frac{2}{\pi}} f(0)$
13.	$f''(x)$		$-\omega^2 \widehat{f}_c(\omega) - \sqrt{\frac{2}{\pi}} f'(0)$
14.	$xf(x)$		$\left[ \widehat{f}_s \right]'(\omega)$
15.	$\mathcal{F}_c(\mathcal{F}_c f)$		$f$

Table of Fourier Sine Transforms

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \mathcal{F}_s(f)(\omega) \sin \omega x \, d\omega,$$

$$0 < x < \infty$$

$$\mathcal{F}_s(f)(\omega) = \widehat{f}_s(\omega) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin \omega x \, dx,$$

$$0 < \omega < \infty$$

$$1. \quad \begin{cases} 1 & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

$$\sqrt{\frac{2}{\pi}} \frac{1 - \cos a\omega}{\omega}$$

$$2. \quad e^{-ax}, \quad a > 0$$

$$\sqrt{\frac{2}{\pi}} \frac{\omega}{a^2 + \omega^2}$$

$$3. \quad x e^{-ax}, \quad a > 0$$

$$\sqrt{\frac{2}{\pi}} \frac{2a\omega}{(a^2 + \omega^2)^2}$$

$$4. \quad \frac{e^{-ax}}{x}, \quad a > 0$$

$$\sqrt{\frac{2}{\pi}} \tan^{-1} \frac{\omega}{a}$$

$$5. \quad \frac{1}{2} x e^{-ax^2}, \quad a > 0$$

$$\frac{\omega}{a^{3/2}} e^{-\omega^2/2a}$$

$$6. \quad \cos ax e^{-ax}, \quad a > 0$$

$$\sqrt{\frac{2}{\pi}} \frac{\omega^3}{4a^4 + \omega^4}$$

$$7. \quad \sin ax e^{-ax}, \quad a > 0$$

$$\sqrt{\frac{2}{\pi}} \frac{2a^2\omega}{4a^4 + \omega^4}$$

$$8. \quad \frac{x}{a^2 + x^2}, \quad a > 0$$

$$\sqrt{\frac{\pi}{2}} e^{-a\omega}$$

$$9. \quad x^{p-1}, \quad 0 < p < 1$$

$$\sqrt{\frac{2}{\pi}} \frac{\Gamma(p) \cos(\pi p/2)}{\omega^p}$$

$$10. \quad \begin{cases} \sin x & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{1}{\sqrt{2\pi}} \left[ \frac{\sin a(1-\omega)}{1-\omega} - \frac{\sin a(1+\omega)}{1+\omega} \right]$$

### Operational Properties

$$11. \quad \alpha f(x) + \beta g(x)$$

$$\alpha \mathcal{F}_s(f)(\omega) + \beta \mathcal{F}_s(g)(\omega)$$

$$12. \quad f(ax), \quad a > 0$$

$$\frac{1}{a} \widehat{f}_s\left(\frac{\omega}{a}\right)$$

$$13. \quad f'(x)$$

$$-\omega \widehat{f}_c(\omega)$$

$$14. \quad f''(x)$$

$$-\omega^2 \widehat{f}_s(\omega) + \sqrt{\frac{2}{\pi}} \omega f(0)$$

$$15. \quad xf(x)$$

$$-[\widehat{f}_c]'(\omega)$$

$$16. \quad \mathcal{F}_s(\mathcal{F}_s f)$$

$$f$$

Table of Laplace Transforms

$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^{\infty} f(t)e^{-st} dt,$
1. 1	$\frac{1}{s}, \quad s > 0$
2. $t^n, \quad n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
3. $t^a \quad (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}, \quad s > 0$
4. $e^{at}$	$\frac{1}{s-a}, \quad s > a$
5. $t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
6. $\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}, \quad s > \max(a, b)$
7. $\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}, \quad s > \max(a, b)$
8. $\sin kt$	$\frac{k}{s^2+k^2}, \quad s > 0$
9. $\cos kt$	$\frac{s}{s^2+k^2}, \quad s > 0$
10. $e^{at} \sin kt$	$\frac{k}{(s-a)^2+k^2}, \quad s > a$
11. $e^{at} \cos kt$	$\frac{s-a}{(s-a)^2+k^2}, \quad s > a$
12. $t \sin kt$	$\frac{2ks}{(s^2+k^2)^2}, \quad s > 0$
13. $t \cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}, \quad s > 0$
14. $\frac{1}{2a^3}(\sin at - at \cos at)$	$\frac{1}{(s^2+a^2)^2}, \quad s > 0$
15. $\sinh kt$	$\frac{k}{s^2-k^2}, \quad s >  k $
16. $\cosh kt$	$\frac{s}{s^2-k^2}, \quad s >  k $
17. $e^{at} \sinh kt$	$\frac{k}{(s-a)^2-k^2} \quad s > a +  k $
18. $e^{at} \cosh kt$	$\frac{s-a}{(s-a)^2-k^2} \quad s > a +  k $
19. $t \sinh kt$	$\frac{2ks}{(s^2-k^2)^2}, \quad s >  k $
20. $t \cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}, \quad s >  k $
21. $\frac{1}{2k^3}(kt \cosh kt - \sinh kt)$	$\frac{1}{(s^2-k^2)^2}, \quad s >  k $

Table of Laplace Transforms (continued)

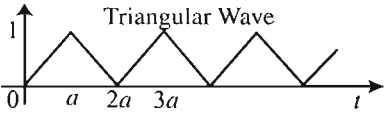
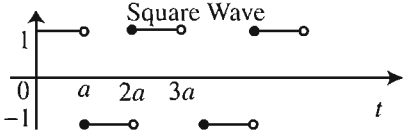
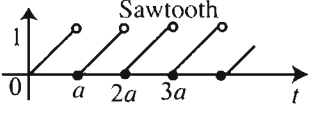
$f(t), t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st} dt,$
22. $\delta_0(t - t_0), t_0 \geq 0$	$e^{-t_0 s}, s > 0$
23. $\mathcal{U}_0(t - a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases} \quad (a > 0)$	$\frac{e^{-as}}{s}, s > 0$
24. $f(t + T) = f(t) \quad (T > 0)$	$\frac{1}{1 - e^{-Ts}} \int_0^T e^{-st} f(t) dt$
25. $f(t + T) = -f(t) \quad (T > 0)$	$\frac{1}{1 + e^{-Ts}} \int_0^T e^{-st} f(t) dt$
26. 	$\frac{1}{as^2} \left[ \frac{1 - e^{-as}}{1 + e^{-as}} \right] = \frac{1}{as^2} \tanh\left(\frac{as}{2}\right), s > 0$
27. 	$\frac{1}{s} \left[ \frac{1 - e^{-as}}{1 + e^{-as}} \right] = \frac{1}{s} \tanh\left(\frac{as}{2}\right), s > 0$
28. 	$\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}, s > 0$
29. $\frac{\sin at}{t}$	$\tan^{-1}\left(\frac{a}{s}\right), s > 0$
30. $J_0(at)$	$\frac{1}{\sqrt{s^2 + a^2}}, s > 0$
31. $J_0(a\sqrt{t})$	$\frac{e^{-a^2/4s}}{s}, s > 0$
32. $t^p J_p(at) \quad (p > -\frac{1}{2})$	$\frac{2^p a^p \Gamma(p + \frac{1}{2})}{\sqrt{\pi}(s^2 + a^2)^{p + \frac{1}{2}}}, s > 0$
33. $\frac{\sqrt{\pi}}{\Gamma(k)} \left(\frac{t}{2a}\right)^{k - \frac{1}{2}} J_{k - \frac{1}{2}}(at) \quad (k > 0)$	$\frac{1}{(s^2 + a^2)^k}, s > 0$
34. $\frac{\sqrt{\pi}}{\Gamma(k)} a \left(\frac{t}{2a}\right)^{k - \frac{1}{2}} J_{k - \frac{3}{2}}(at) \quad (k > \frac{1}{2})$	$\frac{s}{(s^2 + a^2)^k}, s > 0$
35. $2 \sum_{m=1}^n \binom{2n - m - 1}{n - 1} \frac{t^{m-1} \cos(at - \frac{m\pi}{2})}{(2a)^{2n - m} (m - 1)!}$ ( $n$ an integer $\geq 1$ )	$\frac{1}{(s^2 + a^2)^n}, s > 0$
36. $\frac{1}{(n - 1)!} \sum_{m=1}^{n-1} \frac{(2n - m - 3)!}{(m - 1)!(n - m - 1)!} \frac{t^m \cos(at - \frac{m\pi}{2})}{(2a)^{2n - m - 2}}$ ( $n$ an integer $\geq 2$ )	$\frac{s}{(s^2 + a^2)^n}, s > 0$

Table of Laplace Transforms (continued)

$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^{\infty} f(t)e^{-st} dt,$
37. $\operatorname{erf}(at) \quad (a > 0)$	$\frac{1}{s} e^{s^2/4a^2} \operatorname{erfc}\left(\frac{s}{2a}\right), \quad s > 0$
38. $\operatorname{erf}(a\sqrt{t})$	$\frac{a}{s\sqrt{s+a^2}}, \quad s > 0$
39. $e^{-a^2 t^2} \quad (a > 0)$	$\frac{\sqrt{\pi}}{2a} e^{s^2/4a^2} \operatorname{erfc}\left(\frac{s}{2a}\right), \quad s > 0$
40. $\frac{1}{\sqrt{\pi t}} e^{-a^2/4t} \quad (a \geq 0)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}, \quad s > 0$
41. $\frac{a}{2\sqrt{\pi t^{3/2}}} e^{-a^2/4t} \quad (a > 0)$	$e^{-a\sqrt{s}}, \quad s > 0$
42. $\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right) \quad (a \geq 0)$	$\frac{1}{s} e^{-a\sqrt{s}}, \quad s > 0$
<b>Operational Properties</b>	
43. $\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
44. $f'(t)$	$sF(s) - f(0)$
45. $f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
46. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$
47. $-tf(t)$	$F'(s)$
48. $t^n f(t)$	$(-1)^n F^{(n)}(s)$
49. $\int_0^t f(\tau) d\tau$	$\frac{1}{s} F(s), \quad s > 0$
50. $\int_0^t \int_0^\tau f(\rho) d\rho d\tau$	$\frac{1}{s^2} F(s), \quad s > 0$
51. $\frac{f(t)}{t}$	$\int_s^{\infty} F(u) du$
52. $\frac{f(t)}{t^2}$	$\int_s^{\infty} \int_\sigma^{\infty} F(u) dud\sigma$
53. $\mathcal{U}(t-a)f(t-a) \quad (a > 0)$	$e^{-as} F(s)$
54. $e^{at} f(t)$	$F(s-a)$
55. $f(ct) \quad (c > 0)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
56. $f * g(t) = \int_0^t f(\tau)g(t-\tau) d\tau$	$F(s)G(s)$